Tsunami propagation up Columbia river

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$$\frac{\partial h}{\partial t} + div\left(h\overrightarrow{V}\right) = 0$$

$$\left(\frac{\partial}{\partial t} + \overrightarrow{V}\cdot\bigtriangledown\right)\overrightarrow{V} + g\bigtriangledown h + \gamma\frac{g|V|\overrightarrow{V}}{h^{4/3}} = g\bigtriangledown d$$

The Method Of Splitting Tsunami (MOST) numerical model

* was designed to solve SWE efficiently by reducing a 2D problem in space to a sequence of 1D problems;

* solves them indeed when dt sqrt(gd)/dx = 0.999... . Otherwise, solves SWE for the well resolved component of the numerical solution, and

* displays distinct numerical dispersion, which can be used to mimic physical dispersion.

* adapted by NOAA Center for Tsunami Research (NCTR) for tsunami simulations.

Splitting by setting either $\partial/\partial x = 0$ or $\partial/\partial y = 0$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x_1}(hV_1) = 0$$

$$\frac{\partial V_1}{\partial t} + V_1\frac{\partial V_1}{\partial x_1} + g\frac{\partial h}{\partial x_1} + \gamma\frac{g\sqrt{V_1^2+V_2^2}}{h^{4/3}}V_1 = g\frac{\partial d}{\partial x_1}$$

$$\frac{\partial V_2}{\partial t} + V_1\frac{\partial V_2}{\partial x_1} = 0$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x_2}(hV_2) = 0$$

$$\frac{\partial V_2}{\partial t} + V_2\frac{\partial V_2}{\partial x_2} + g\frac{\partial h}{\partial x_2} + \gamma\frac{g\sqrt{V_1^2+V_2^2}}{h^{4/3}}V_2 = g\frac{\partial d}{\partial x_2}$$

$$\frac{\partial V_1}{\partial t} + V_2\frac{\partial V_1}{\partial x_2} = 0$$

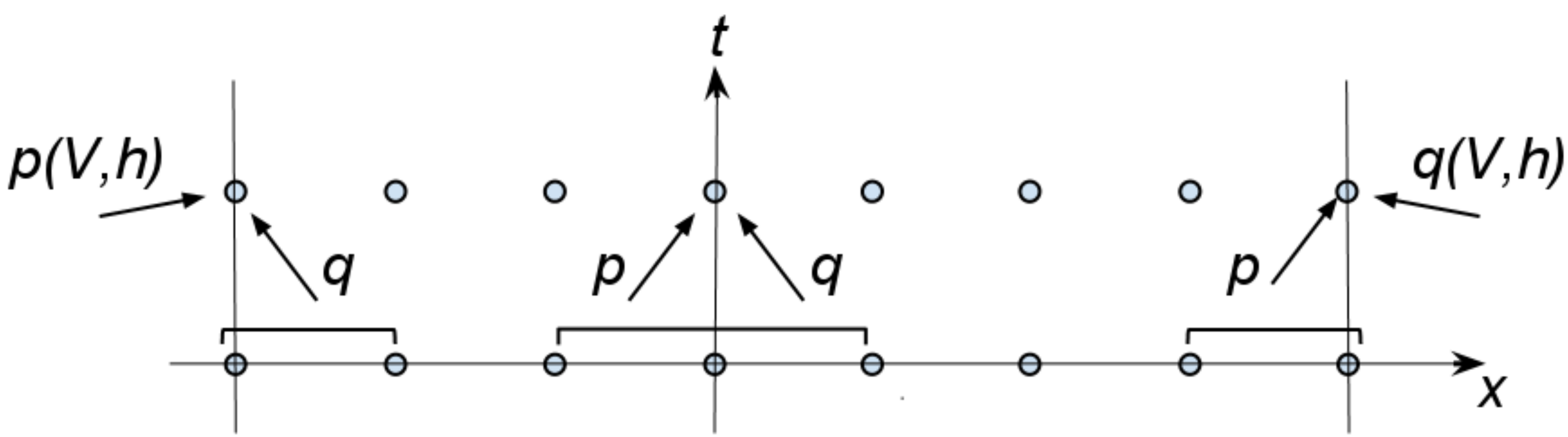
Solved in terms of Riemann invariants $p = V + 2\sqrt{gh}, \quad q = V - 2\sqrt{gh}$
and eigenvalues: $\lambda_{p,q} = V \pm \sqrt{gh}$

Forward difference in dt and 2-nd order approximation in dx (except on boundaries):

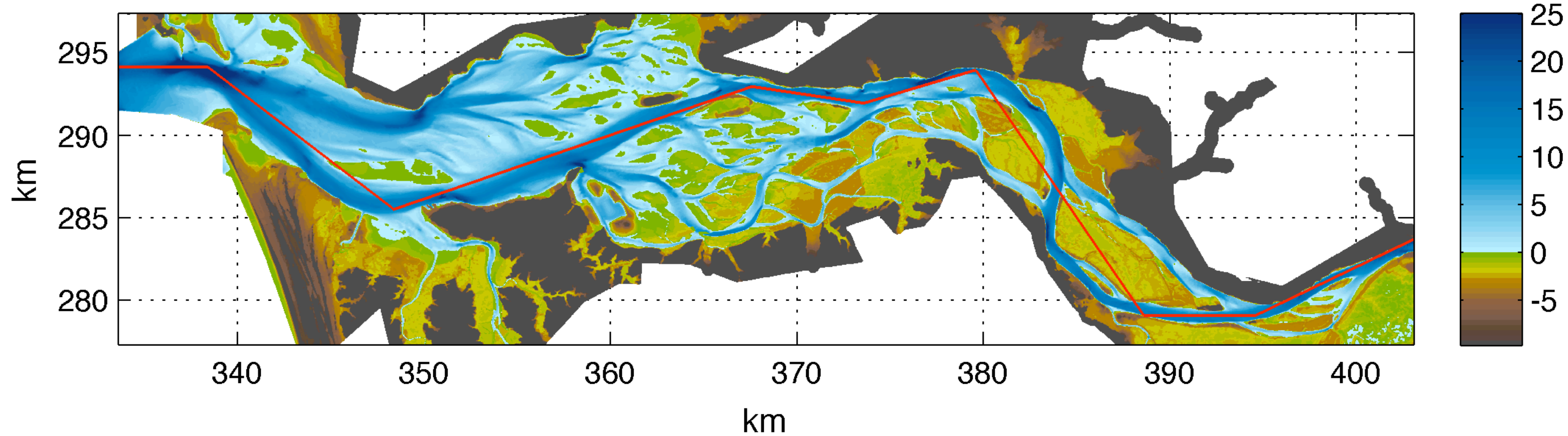
$$p_j^{n+1} = p_j + \frac{\Delta t}{2\Delta x}\left[-(\frac{\lambda_{j+1} + \lambda_{j-1}}{2})(p_{j+1} - p_{j-1}) + g(d_{j+1} - d_{j-1})\right] +$$
$$\frac{\Delta t^2}{2\Delta x^2}\left[-g\lambda_j(d_{j+1} + d_{j-1} - 2d_j) + \frac{\lambda_j}{2}\{(\lambda_{j+1} + \lambda_j)(p_{j+1} - p_j) - (\lambda_j + \lambda_{j-1})(p_j - p_{j-1})\}\right]$$

Primitive variables recovered as $V = (p+q)/2, \quad h = (p-q)^2/16g.$

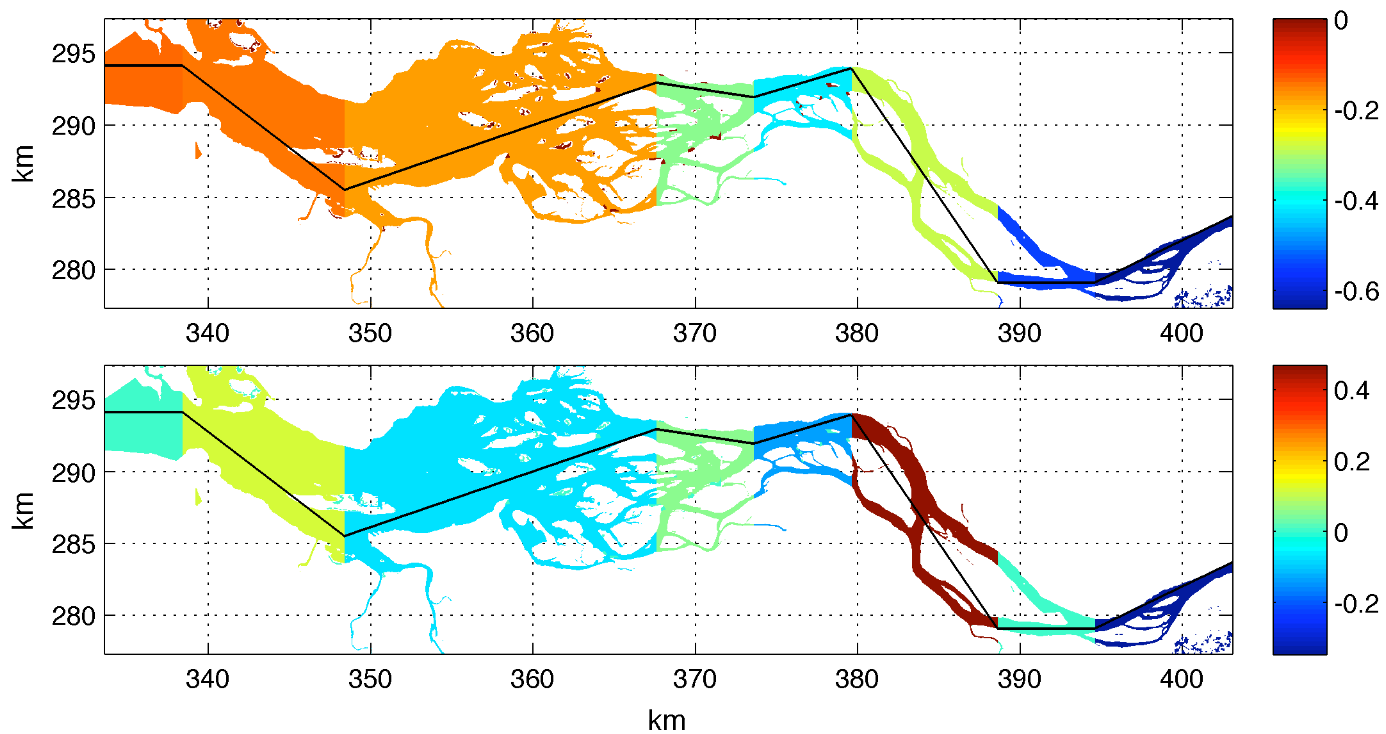
Boundary conditions: totally reflective and totally transparent. Incoming Riemann invariant is computed with ‘outside’ current and elevation.



Columbia River basin



Piece-wise constant initial velocities (top: WestEast, bottom: SouthNorth, m/s)



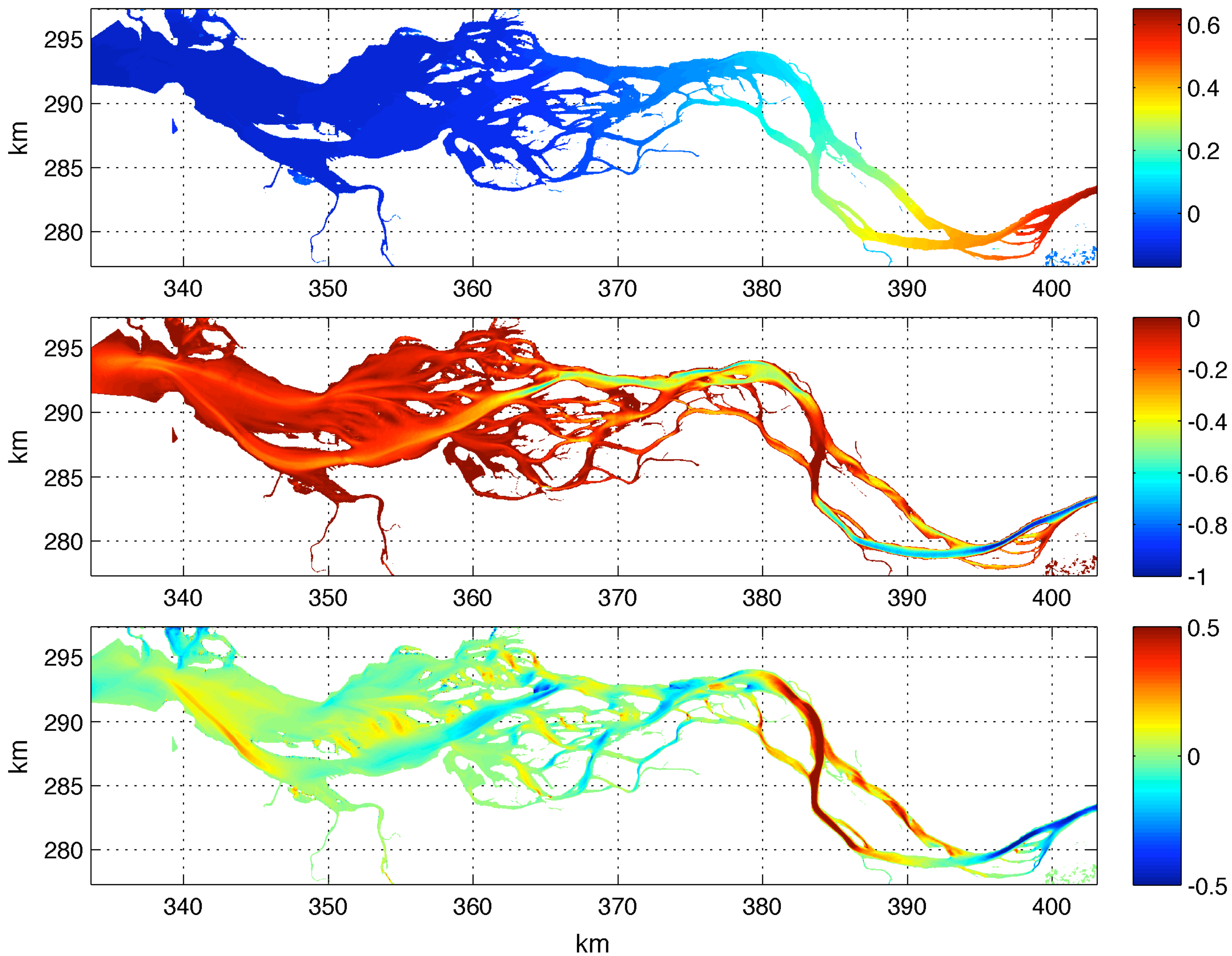
Iterations to simulate river's current:

Simulate the river flow for several hours with given initial/boundary conditions.

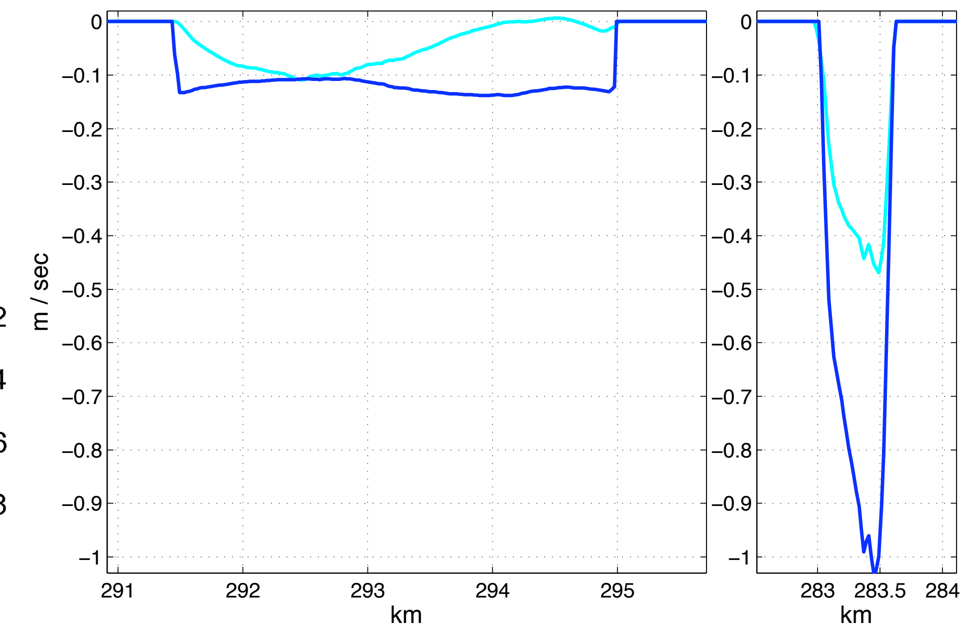
Replace initial conditions with the results of the simulation. Replace boundary conditions with solution on the boundaries, scaled to provide the given discharge.

Repeat, until steady state is reached.

Hydraulic lead (top), WestEast (middle) and SouthNorth (bottom) current in the steady flow



Horizontal (blue) and vertical (cyan) currents on the left and right boundaries

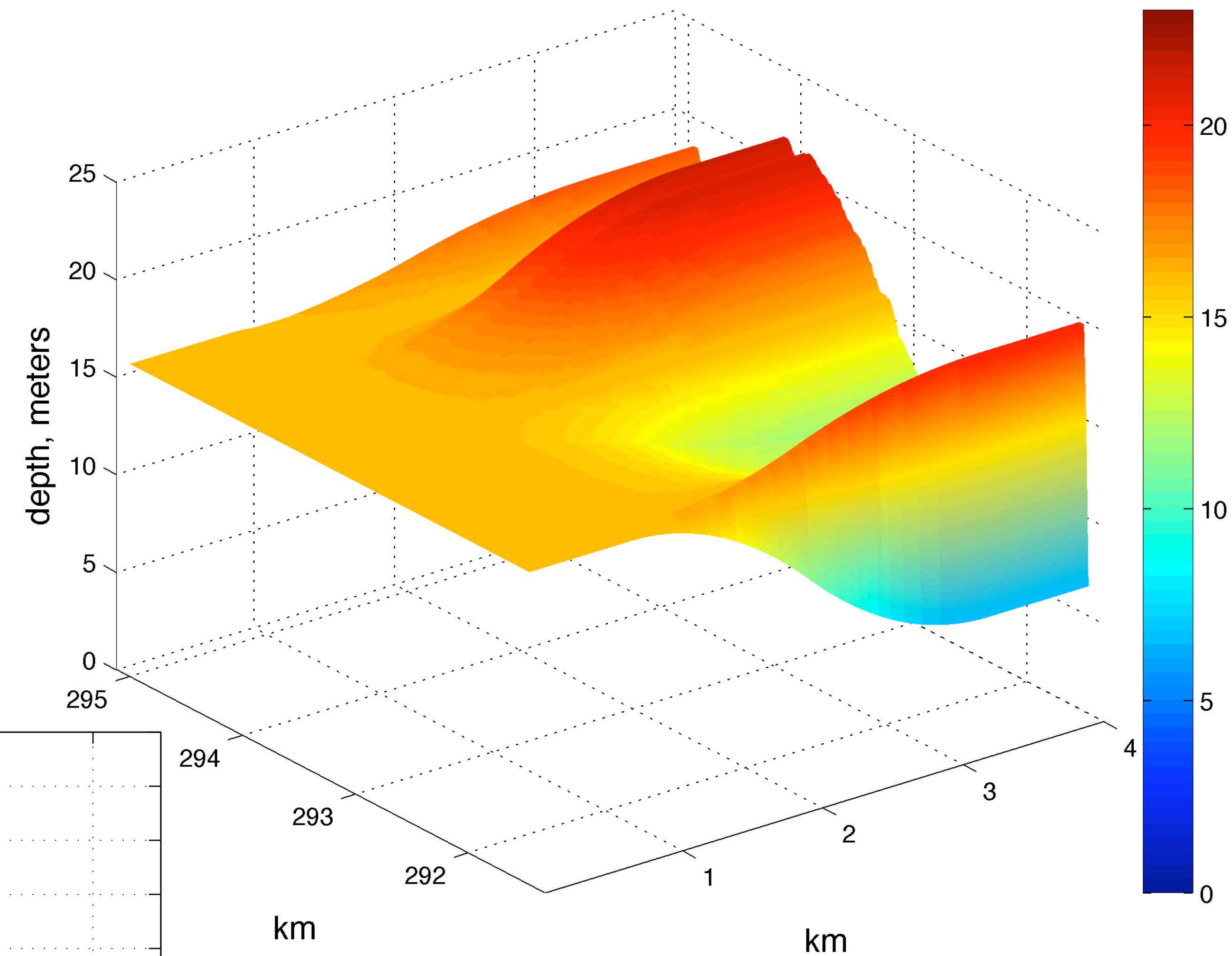


Change in elevation: -0.13 m on the left, 0.64 m on the right, head loss 0.77 m.

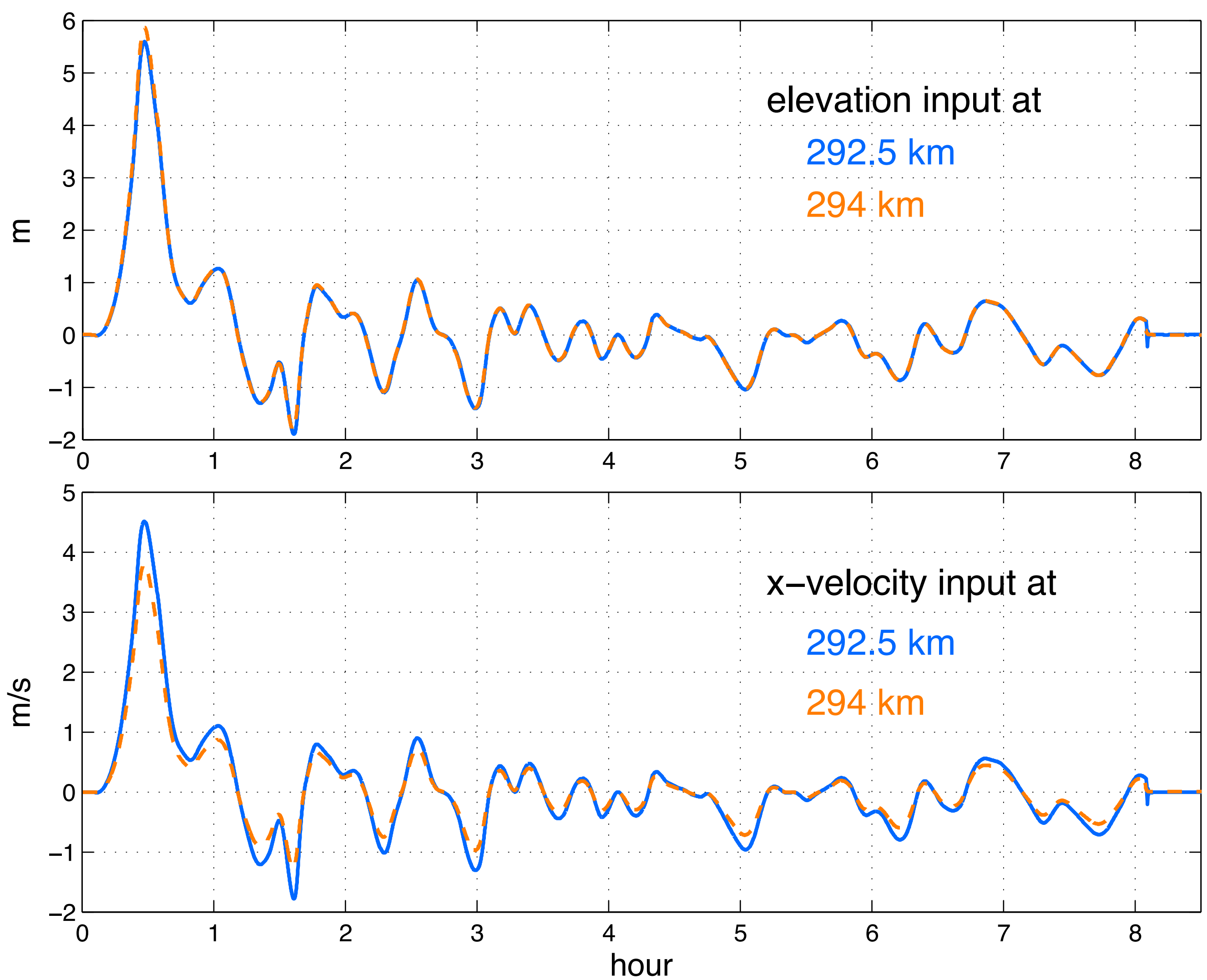
Estimate for Darcy-Weisbach head loss $h = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g}$

with the river length L=70 km, hydraulic diameter D=4depth=40 m, average current V=0.6 m/s, and Darcy factor f=0.025, computed h=0.79 m.

Importing the Tsunami



Input to the river from the adaptor's output:



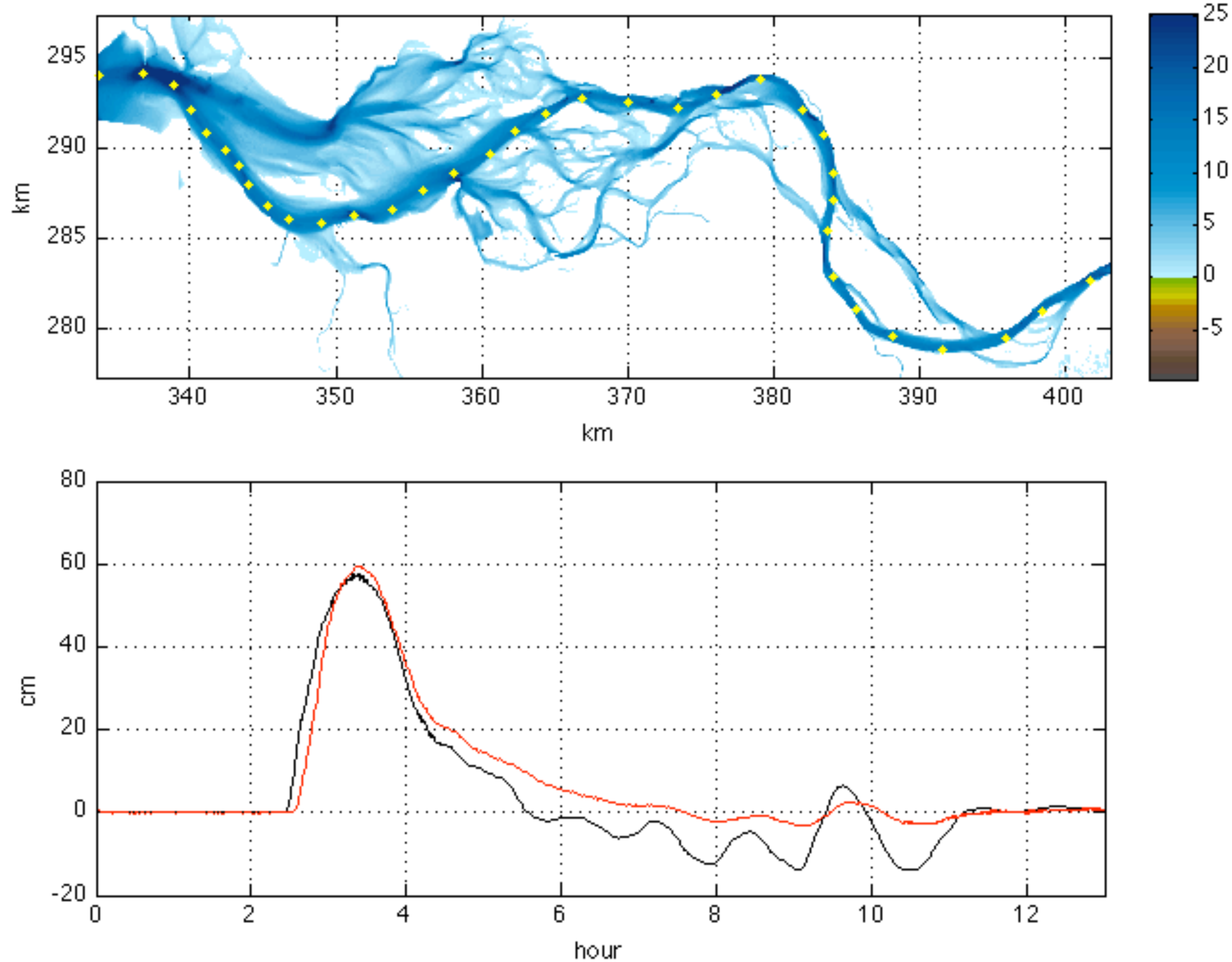
Bathy adaptop:

channel with depth gradually transforming from constant d to following given profile.

Input of elevation $\eta(t)$

and velocity $V(t) = \eta\sqrt{g/(d+\eta)}$

from the flat end.



Movie: Tsunami propagation up Columbia river, time histories at 35 locations in flowing river (red) vs. quiescent river (black). Bottom - location 35.

Tides

Input to simulate tides

river mouth:

$$u(y,t) = \alpha u_o(y) + 1.95\cos(2\pi t/T)$$

$$v(y,t) = 0$$

$$\eta(y,t) = 0$$

upstream:

$$u(y,t) = u_{up} \cdot (\beta + b_1\cos(\omega t) + b_2\sin(\omega t))$$

$$v(y,t) = 0$$

$$\eta(y,t) = 0$$

coefficients selected for given discharge: $\alpha = 3.95,\ \beta = 1.4$

Tides modify the river's own discharge.

Given the height of water column, accounting for tides,

$$h(t) = h_0 + a\cos(\omega t + \phi)$$

and the current

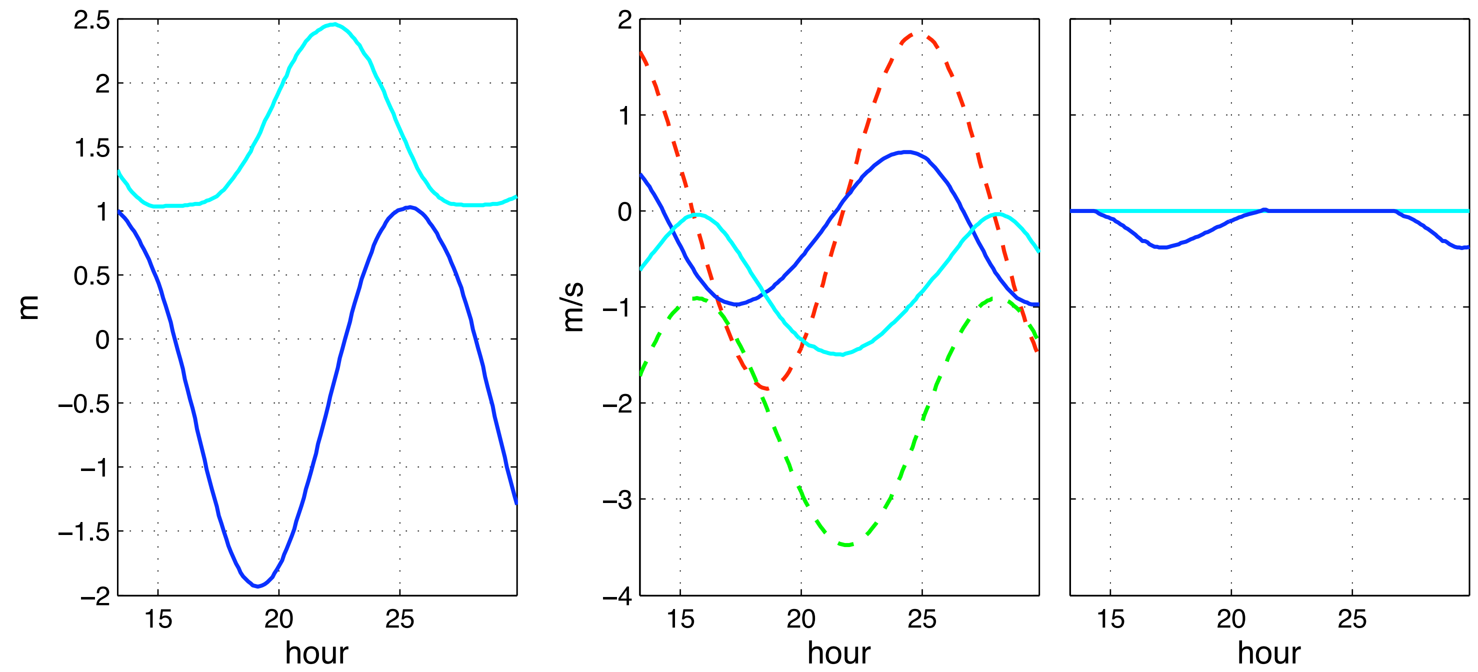
$$u(t) = u_0 + b\cos(\omega t),$$

the river's discharge averaged over the tidal period is

$$\frac{1}{T}\int_0^T u(t)h(t)dt = u_0 h_0 + \frac{1}{2}ab\cos\phi,$$

which is different from the river's discharge without tides $u_0 h_0$.

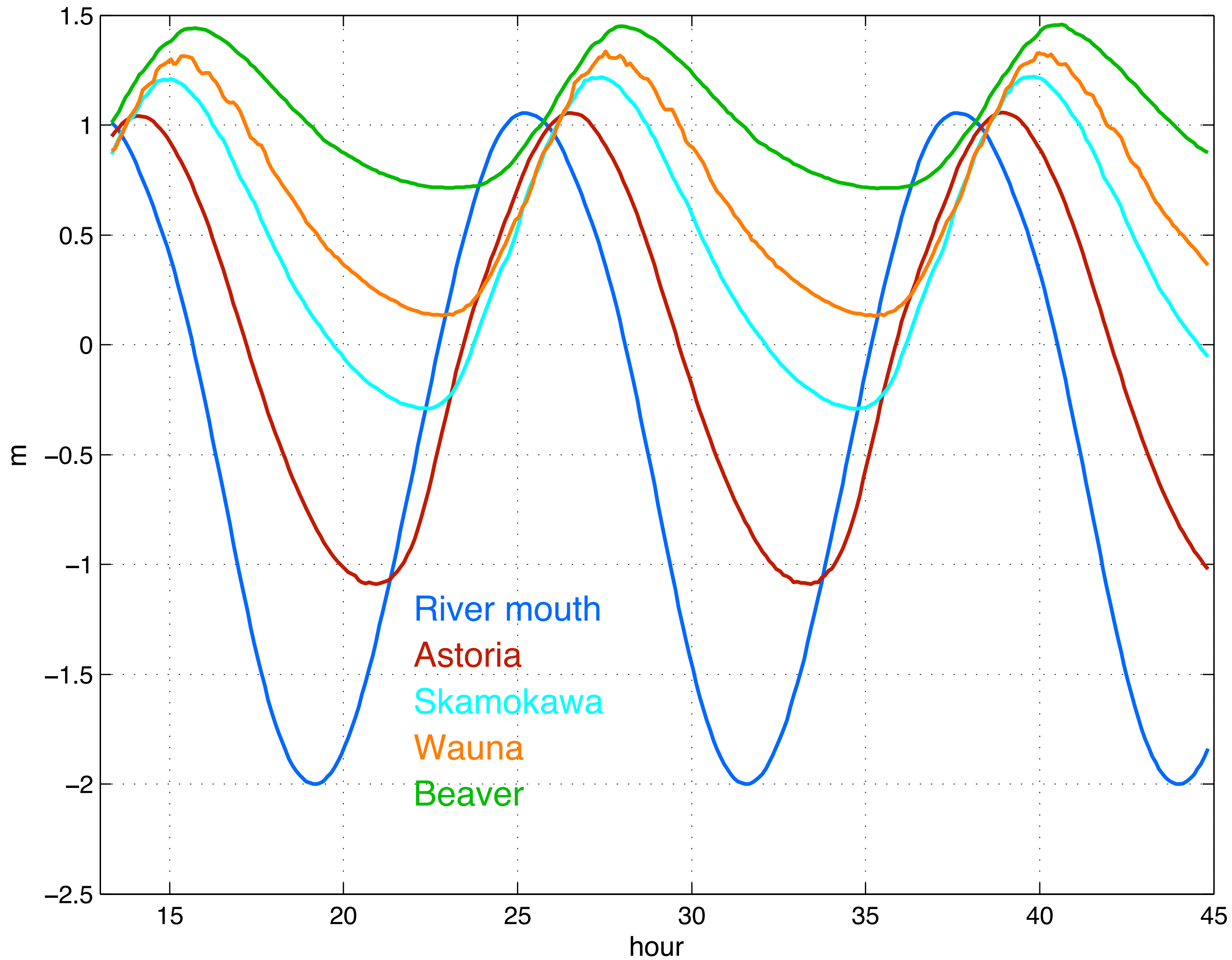
This approach relies on the model's ability to extract the true solution from a cocktail fed into the model.



Sample model solution on the left (ocean) boundary (blue) and the right (upstream) boundary (cyan) for elevation (left pane), x-current (middle), y-current (right). X-current input on the left (red dashed) and right (green dashed).

To simulate tsunami atop tides, the tsunami boundary input was combined with the tidal solution on boundaries.

Simulated tidal records at locations



Model over-estimated reduction in the tidal range with distance.

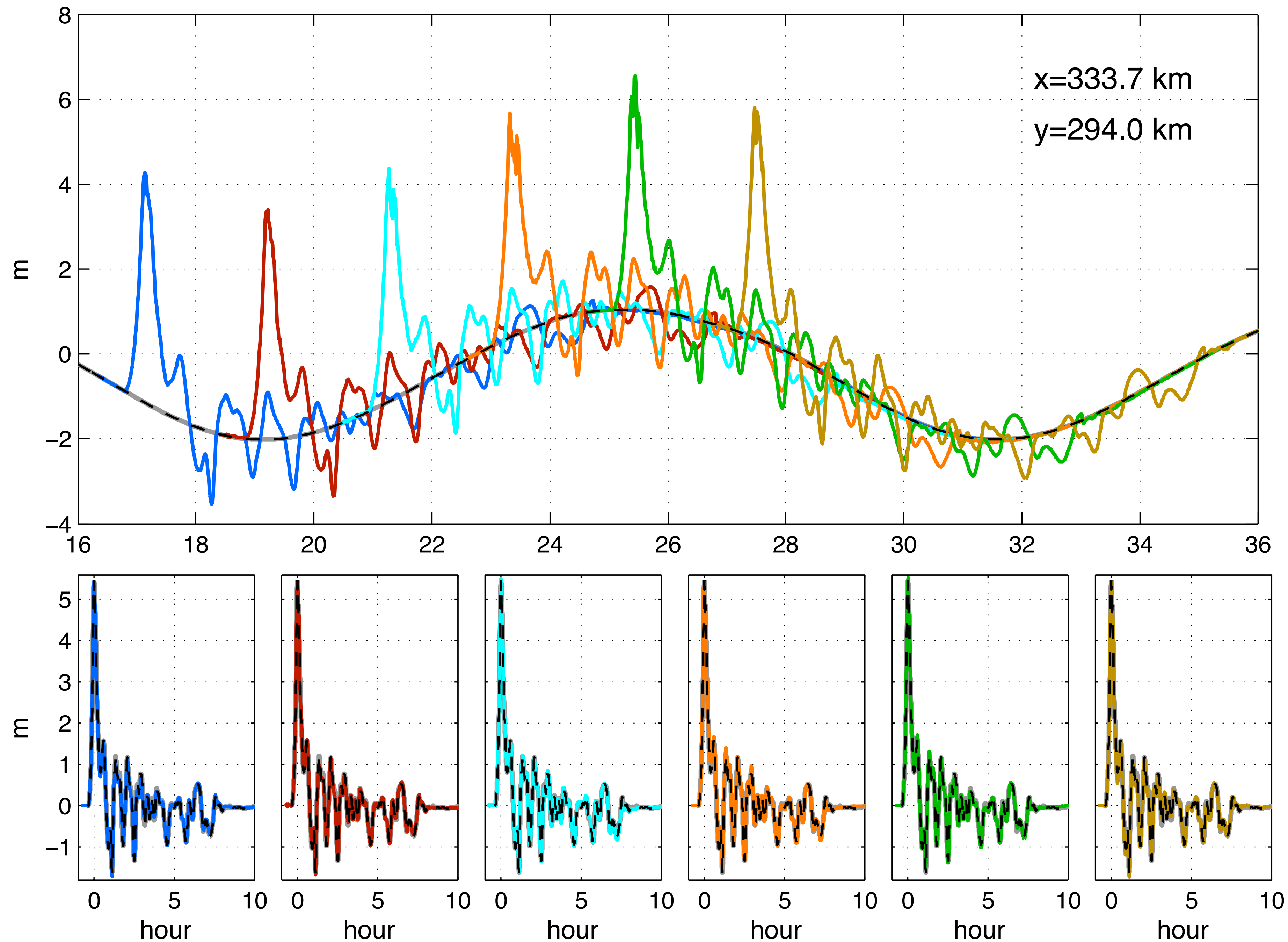
Astoria to Wauna: reduction 1.2 times in data vs. 1.8 in the model.

River mouth to Beaver: reduction 4.1 times in model.

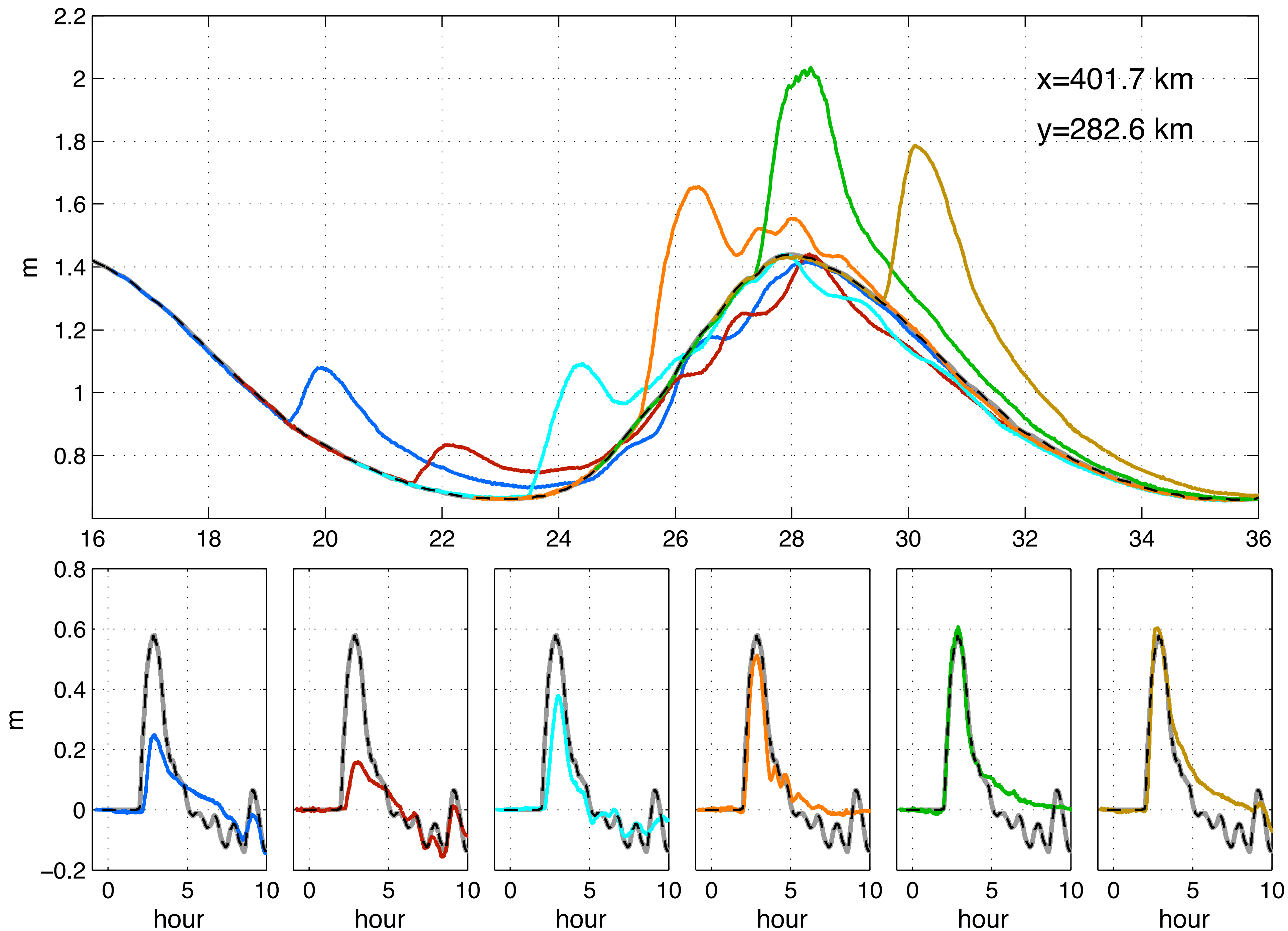
Should probably be 4.1 x 1.2 / 1.8=2.7 times, that is, 1 m tidal range in Beaver for 3 m range at the mouth.

	M2 amplitude, m		M2 time lag with Astr, h	
	data	model	data	model (±0.1h)
River mouth		1.53		-1.3
Astoria	0.945	1.07		0
Skamokawa	0.846	0.75	0.95	0.9
Wauna	0.79	0.60	1.25	1.2
Beaver		0.37		1.5

Movie: Six different model runs for tsunamis coming at different tidal phase, overlaid in the top pane. Bottom: de-tided time-series from the top pane, vs. time-series of the reference tsunami (black-dashed) propagating in the still (no tides, no flow) river.



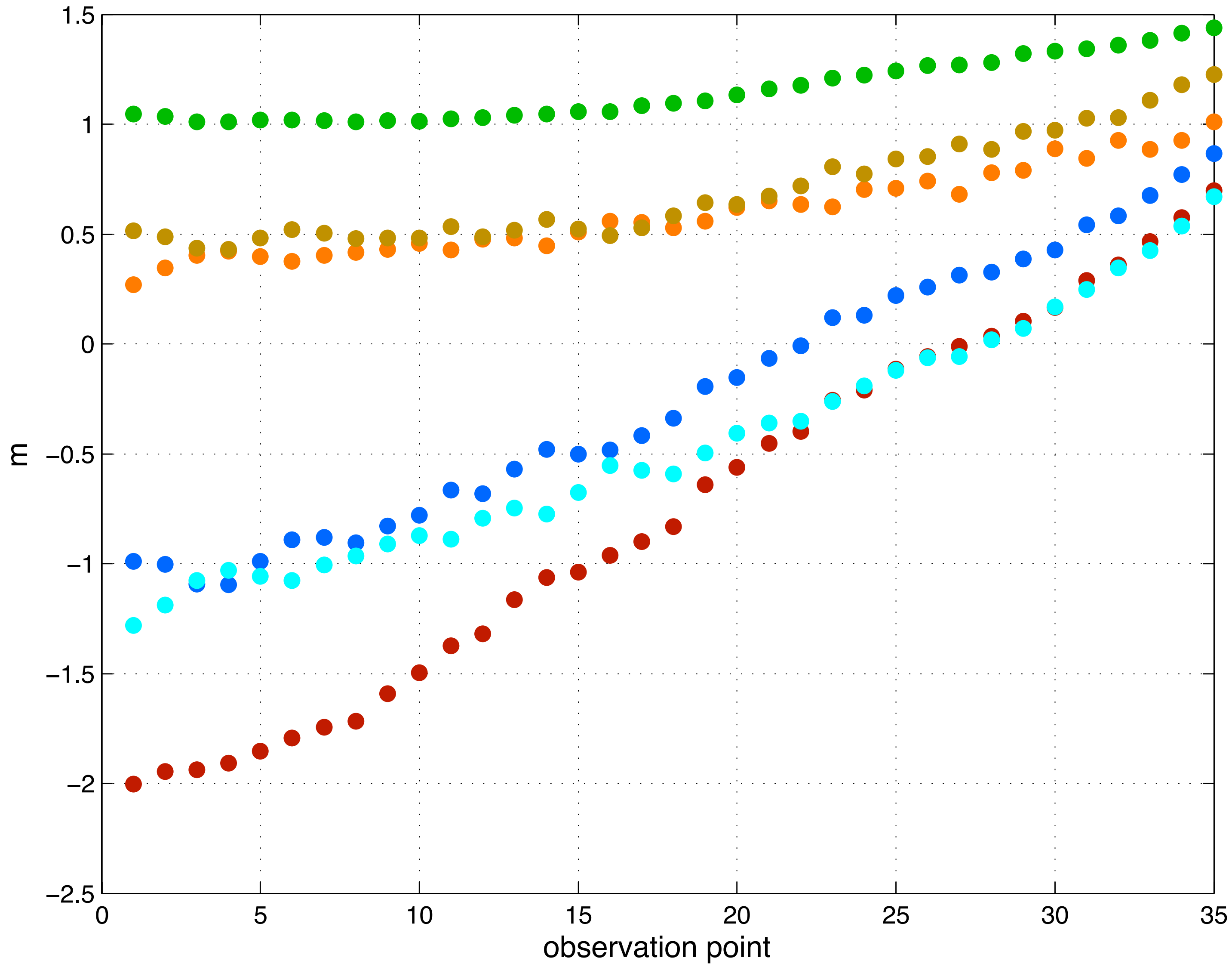
Location 1: river mouth

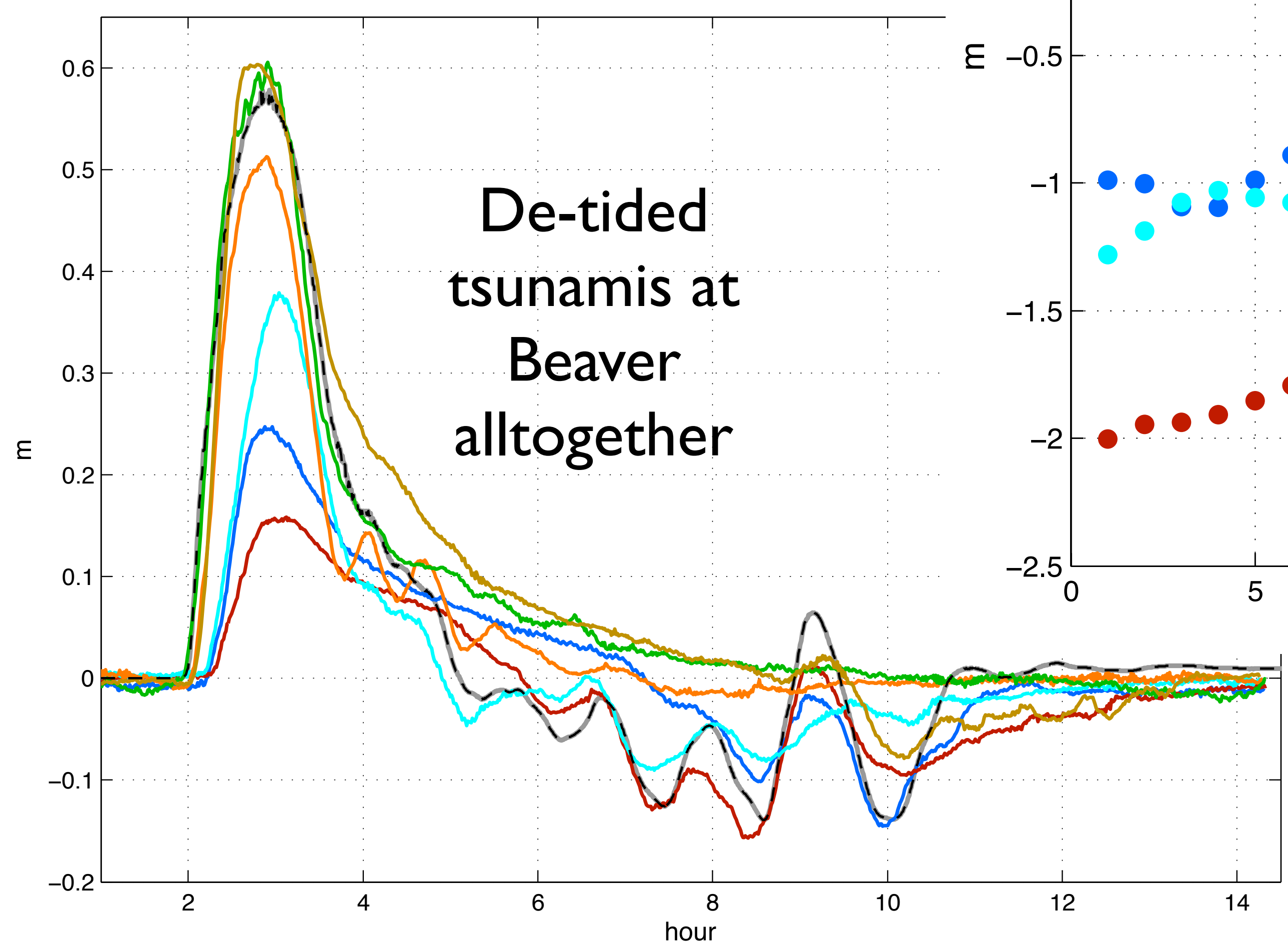


Location 35: near Beaver

Tidal elevation at each location taken at the moment of tsunami passage. Colors correspond to tsunamis riding specific tidal phases, eg:

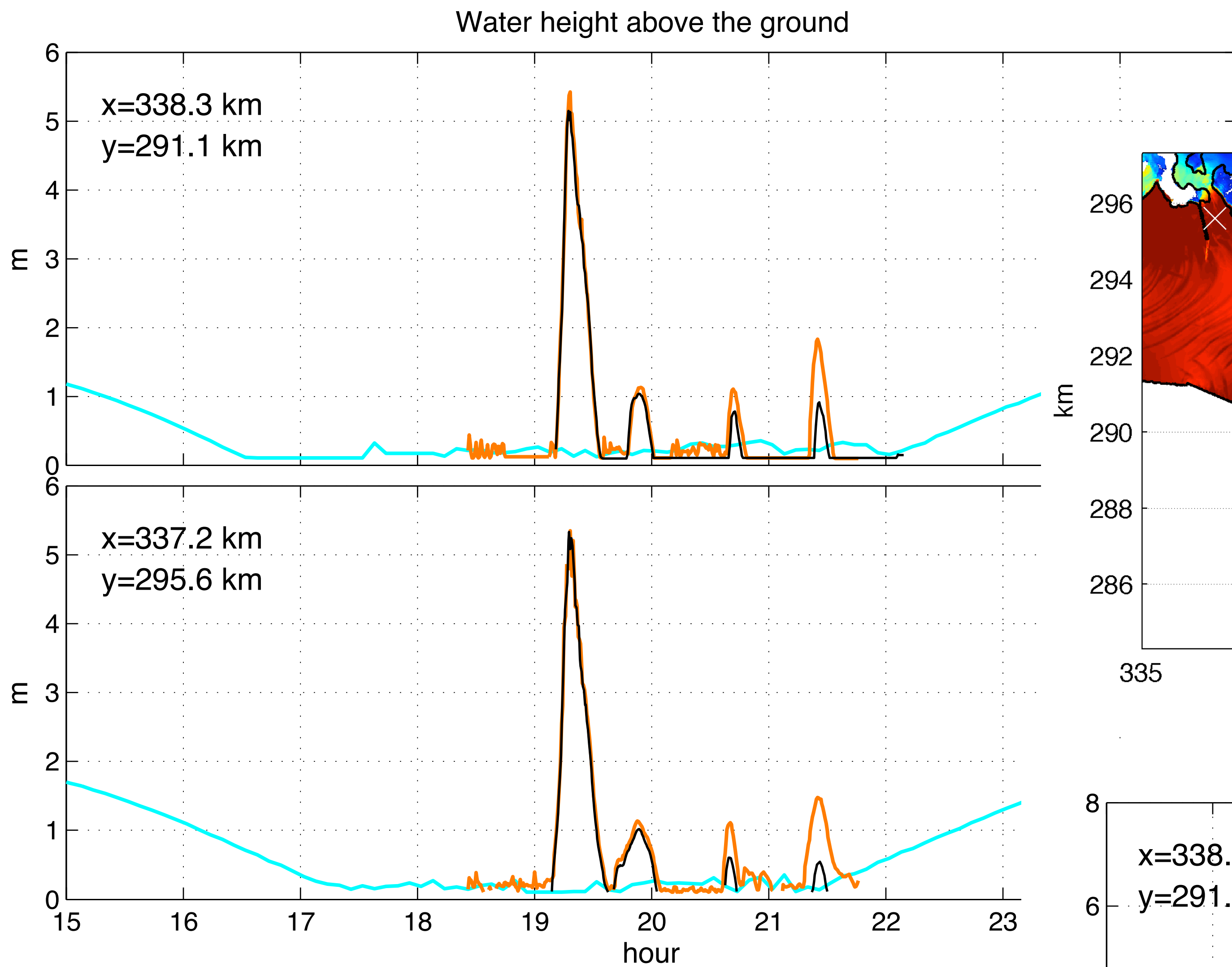
green - tidal maxima at the locations; >
orange - tidal elevation T/6 before maximum at each location (T - tidal period); > ...

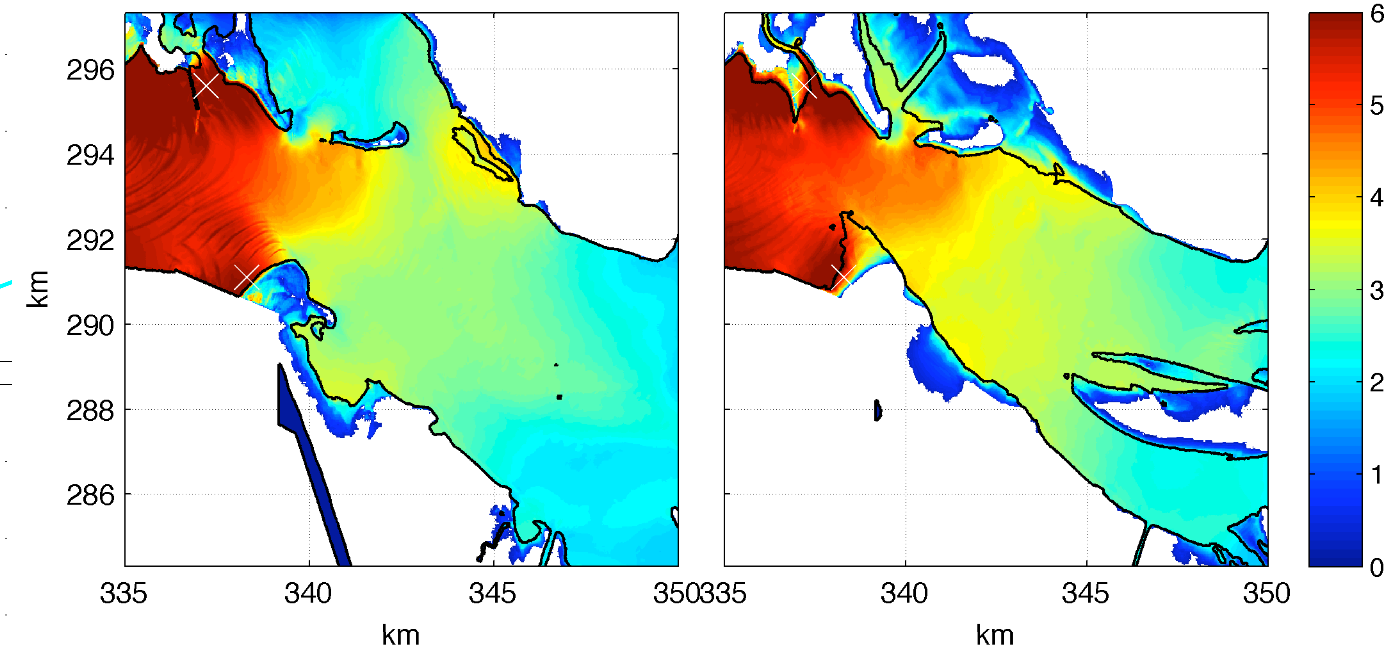


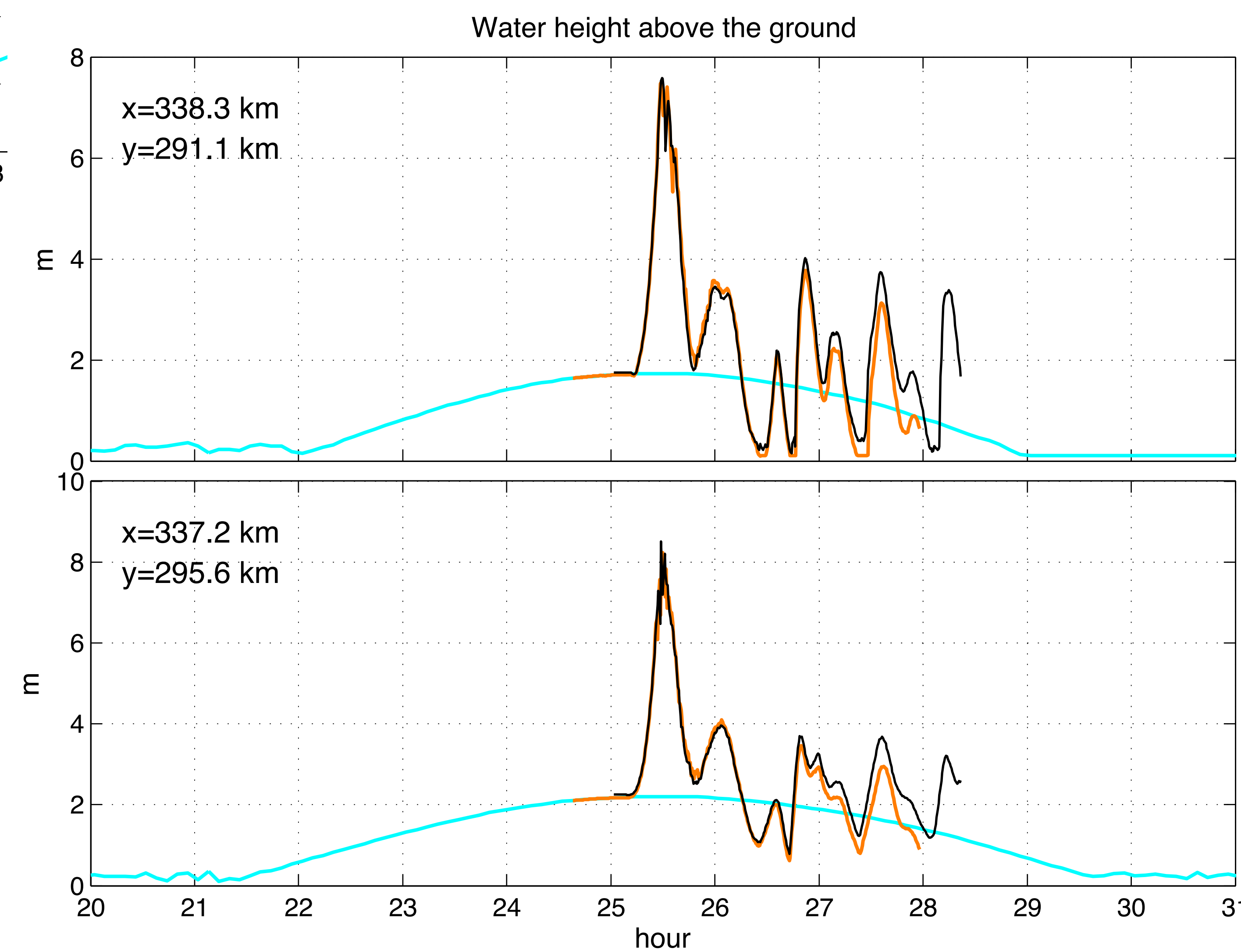


<u>Remarks:</u> constant tidal range = still basin (since tsunami wave keeps its position relative to tidal wave).
Change in tidal range > tsunami goes uphill/downhill > loss/gain of energy.

Inundation







Max wave height from the still water surface / land. Black - river limits at high/low tide.

Time histories: tide (cyan), tsunami + tide (orange), tsunami in quiescent river of matching depth (black).

Base depth: 1.2 m (North gage), 0.7 m (South gage).