THE THIRD INTERNATIONAL WORKSHOP ON LONG-WAVE RUNUP MODELS

June 17-18 2004
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June/2004

REPORT

BENCHMARK PROBLEM -1- (BM1)

Tsunami runup onto a plane beach

1. Problem Description

This is a simple setup for tsunami runup modeling exercise: a uniformly sloping beach and no variation in the lateral direction, viz. a 2-D problem in the vertical plane. The initial-value-problem (IVP) technique introduced by Carrier, Wu and Yeh (Journal of Fluid Mechanics, 475, 79-99, 2003) is used to produce the benchmark data. For the benchmark problem $N.1$, the beach slope is 1/10 and the initial free surface elevation is given. Assignment is to compute and present the snapshots of the free surface and velocity profiles at $t = 160$ sec., 175 sec., and 220 sec. The detailed shoreline trajectory is the primary theme. We describe the algorithm used to calculate the motion of the shoreline (the air-water-beach interface), Specifically, the temporal variations of the shoreline location and shoreline velocity from $t = 100$ sec. to 280 sec., is presented

2. Introduction

To Solve BM1, three approaches have been carried out:

a) First order approximation in time

b) Second order approximation in time (Leap-frog)

c) Volume of Fluid Method VOF

Approaches a) and b) use 1D shallow water wave theory. The finite difference solution of the equation of motion and the depth integrated continuity equation are solved on a staggered grid. Both method possess a second order approximation in space. Case a) has truncation error in time of order one.

A 2D VOF solution has been incorporated to visualize differences between the shallow water theory (SW) and the full solution of Navier-Stokes (N-S). This method models transient, incompressible fluid flow with free surface. The finite difference solution of the incompressible N-S equations are obtained on a Eulerian rectilinear mesh. Friction terms appear in the equations but have been neglected in all cases.

3. Description of the Models

3.1 First Order Method

Equations of motion and continuity read,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \zeta}{\partial x} + \frac{1}{\rho D} ru|u| = 0 \tag{1}$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial uD}{\partial x} = 0 \tag{2}$$

where ρ is the water density, u, is the particle velocity vertically averaged, , H is the mean water depth, ζ is the sea level, $D = (\zeta + H)$ is the total depth, r is the friction coefficient, and g is the gravity acceleration.

The numerical solution of this system is usually searched by using one-time-level or the two-time-level numerical scheme (Kowalik and Murty, 1993a). For construction of the space derivatives in equations (1) and (2), a space staggered grid is used (Arakawa C grid - see fig. 1).

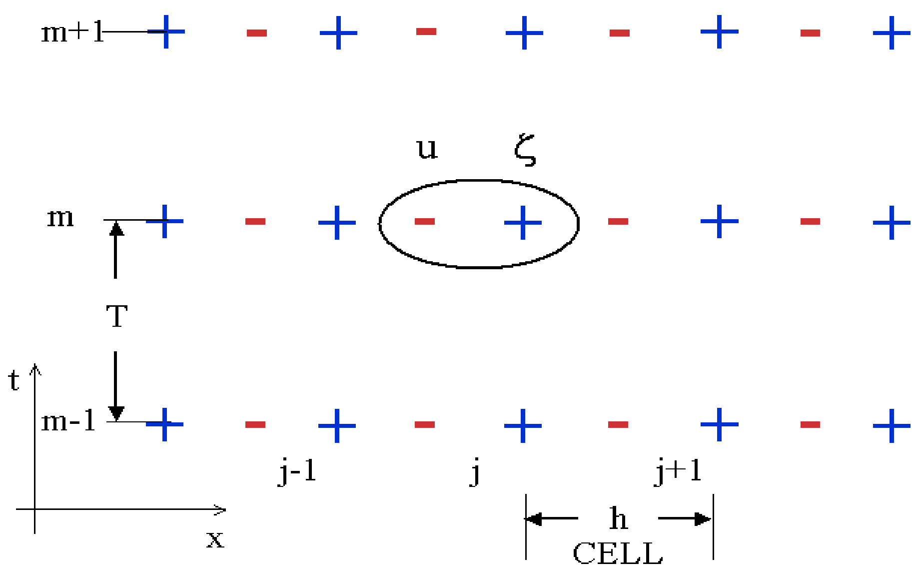

Figure 1. Time-space grid for the 1D problem.

The numerical scheme is constructed as follow

$$u_j^{m+1} = u_j^m - \frac{gT}{h}(\zeta_j^m - \zeta_{j-1}^m) - \frac{u_p^m T}{h}(u_j^m - u_{j-1}^m) - \frac{u_n^m T}{h}(u_{j+1}^m - u_j^m)$$

$$-\frac{2T}{\rho(D_{j-1}^m + D_j^m)} r u^m |u^m| \tag{3}$$

$$\zeta_j^{m+1} = \zeta_j^m - \frac{T}{h}(flux_{j+1}^{m+1} - flux_j^{m+1}) \tag{4}$$

where:

$flux_j = u_p^{m+1}\zeta_{j-1}^m + u_n^{m+1}\zeta_j^m + u_j^{m+1}\frac{(H_j + H_{j-1})}{2}$
$u_p = 0.5 * (u_j + |u_j|)$
$u_n = 0.5 * (u_j - |u_j|)$

and

T is time step, h is space step. Index m and $j = 1, 2, 3, ...n-1, n$ stand for the time stepping and horizontal coordinate points respectively.

A very simple runup condition is used. The following steps are taken when the dry point $j_{wet} - 1$ is located to the left of the wet point j_{wet}:

$$IF(\zeta^m(j_{wet}) > -H(j_{wet} - 1)) \text{ THEN } \zeta^m_{j_{wet}-1} = \zeta^m_{j_{wet}} \text{ and } u^m_{j_{wet}} = u^m_{j_{wet}+1}$$

A Similar approach is used for the dry point located to the right of the wet point, Kowalik and Murty (1993b). This simple extrapolation seems to do a good job in following the shoreline evolution. Results obtained are depicted in figs. 4-7.

3.2 Leap Frog Method

Equations of motion and continuity are expressed in water transport form.

$$\frac{\partial M}{\partial t} + \frac{\partial M^2}{\partial x} + gD\frac{\partial \zeta}{\partial x} + \frac{gn^2}{D^{7/3}}M|M| = 0 \tag{5}$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial M}{\partial x} = 0 \tag{6}$$

where: $M = uD$ is the water transport, n is the Manning's roughness coefficient.

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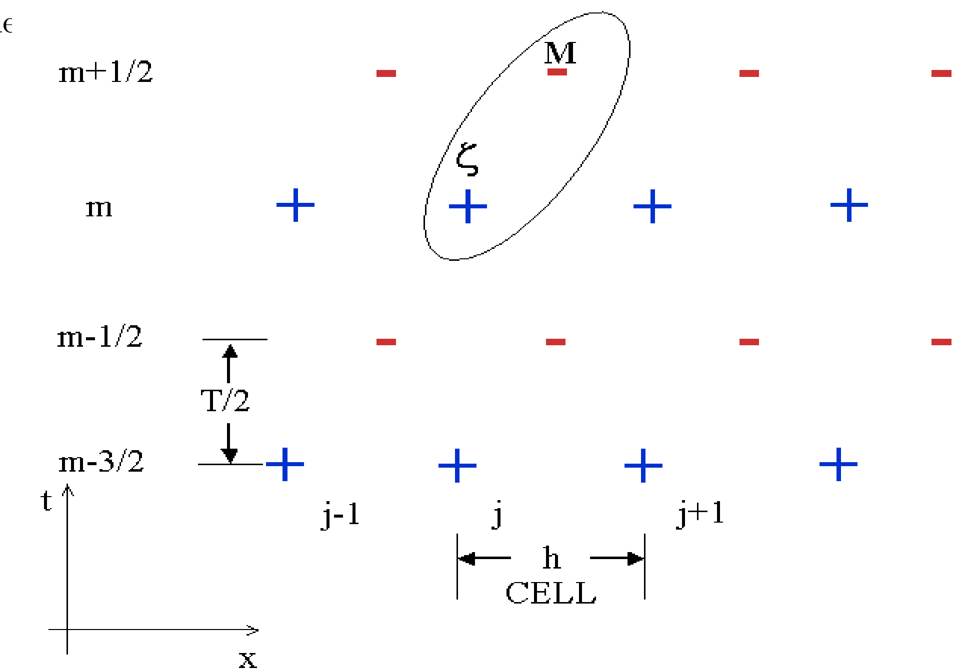

Figure 2. Time-space grid for the 1D Leap-Frog Method.

The numerical scheme is second order of approximation in space and time. The following numerical scheme has been extracted from the work done by Goto, Shuto, Ogawa and Imamura, (1995). The two-time-level numerical scheme follows:

$$\zeta^m_j = \zeta^{m-3/2}_j - \frac{T}{h}(M^{m-1/2}_j - M^{m-1/2}_{j-1}) \tag{7}$$

$$M_j^{m+1/2} = \frac{1}{(1+\mu_x)}[(1-\mu_x)M_j^{m-1/2} - \frac{gD_rT}{h}(\zeta_{j+1}^m - \zeta_j^m)$$

$$-\frac{T}{h}(\lambda_1 \frac{(M_{j+1}^{m-1/2})^2}{D_{M\,j+1}^{\;m-1/2}} + \lambda_2 \frac{(M_j^{m-1/2})^2}{D_{M\,j}^{\;m-1/2}} + \lambda_3 \frac{(M_{j-1}^{m-1/2})^2}{D_{M\,j-1}^{\;m-1/2}})] \tag{8}$$

where:

μ_x is a friction term factor, D_M is the total depth at M points, and D_r is the total depth and depends of the sea level and depth of the neighboring cells.

λ_1, λ_2 and λ_3 are the up-down wind's switches used in the nonlinear term. They are defined as:

If $M_j^{m-1/2} <= 0.0$ and $D_{M\,j+1}^{\;m-1/2} <= 0.0$ Then $\lambda_1 = 0$ and $\lambda_3 = 0$

If $M_j^{m-1/2} <= 0.0$ and $D_{M\,j+1}^{\;m-1/2} > 0.0$ Then $\lambda_1 = 0$ and $\lambda_3 = 1$

If $M_j^{m-1/2} > 0.0$ and $D_{M\,j-1}^{\;m-1/2} <= 0.0$ Then $\lambda_1 = 0$ and $\lambda_3 = 0$

If $M_j^{m-1/2} > 0.0$ and $D_{M\,j-1}^{\;m-1/2} > 0.0$ Then $\lambda_1 = 1$ and $\lambda_3 = 0$

Usually the Leap-Frog scheme has a second order of approximation. However, as long as the advection term concerns, the truncation error is of order one.

μ_x and D_M are defined as:

$\mu_x = \frac{gn^2T}{2D_r^{7/3}}|M_j^{m-1/2}|$,

$D_{M\,j}^{\;m-1/2} = \frac{1}{4}(D_j^{m-3/2} + D_{j+1}^{m-3/2} + D_j^m + D_{j+1}^m)$, recall that $D = (\zeta + H)$ is the total depth at sea level points.

The D_r defines the value that will be used in the momentum equation to calculate the new transport $M_j^{m+1/2}$.

D_r is calculated using the following set of statements:

If $D_j^m <= 0.0$ and $D_{j+1}^m <= 0.0$ then $D_r = 0.0$

If $D_j^m <= 0.0$ and $D_{j+1}^m > 0.0$ then $D_r = max(0.0, \zeta_{j+1}^m + H_j)$

If $D_j^m > 0.0$ and $D_{j+1}^m <= 0.0$ then $D_r = max(0.0, \zeta_j^m + H_{j+1})$

If $D_j^m > 0.0$ and $D_{j+1}^m > 0.0$ then $D_r = 0.5(D_j^m + D_{j+1}^m)$

If $D_r = 0$ then $M_j^{m+1/2}$=0, otherwise calculate $M_j^{m+1/2}$ according to eq. 8.

This method does a good job in predicting shore line evolution as well as the previous one.

Results obtained by the above method are shown in figs. 8-11.

3.3 VOF Method

Equation of continuity for incompressible fluid

$$\nabla \cdot \vec{V} = 0 \tag{9}$$

and the momentum equation,

$$\frac{\partial \vec{V}}{\partial t} + \nabla \cdot (\vec{V}\vec{V}) = -\frac{1}{\rho}\nabla p + \frac{1}{\rho} \cdot \tau + \vec{g} \tag{10}$$

are solved in the rectangular system of coordinate. There: $\vec{V}(x, y, t)$ is the velocity vector, ρ is the fluid density, p is the scalar pressure, τ is the viscous stress tensor, $\vec{g}$ is the acceleration due to gravity and t is the time.

Solution of the above equations is searched using the two-step method, Harlow-Welch (1965), and Chorin (1968). The time discretization of the momentum equation is given by

$$\frac{\vec{V}^{m+1} - \vec{V}^{m}}{T} = -\nabla \cdot (\vec{V}\vec{V})^{m} - \frac{1}{\rho^m}\nabla p^{m+1} + \frac{1}{\rho^m} \cdot \tau^m + \vec{g} \tag{11}$$

and it is broken up into two steps as follow:

$$\frac{\tilde{\vec{V}} - \vec{V}^{m}}{T} = -\nabla \cdot (\vec{V}\vec{V})^{m} + \frac{1}{\rho^m} \cdot \tau^m + \vec{g} \tag{12}$$

$$\frac{\vec{V}^{m+1} - \tilde{\vec{V}}}{T} = -\frac{1}{\rho^m}\nabla p^{m+1} \tag{13}$$

$$\nabla \cdot \vec{V}^{m+1} = 0 \tag{14}$$

In the first step, a velocity filed $\tilde{\vec{V}}$ is computed from $\vec{V}^m$. In the second step, this velocity field is introduced into equation 13. Equation 13 and 14 can be combined into a single Poisson equation for the solution of the pressure as,

$$\nabla \cdot [\frac{1}{\rho^m}\nabla p^{m+1}] = \frac{\nabla \cdot \tilde{\vec{V}}^{m+1}}{T}. \tag{15}$$

The free surfaces of the fluid is described with discrete volume-of-fluid (VOF) method. The VOF method, pioneered by Hirt and Nichols (1975) is a powerful tool that enable a finite difference representation of the free surface and interfaces that are arbitrarily oriented with respect to the computational grid. This method has been extensively used for prediction of runup, see work done by Mader (1999).

The VOF function is advected as a Lagrangian invariant, propagating according to

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + (\vec{V} \cdot \nabla)F = 0 \tag{16}$$

and is the only avaliable free surface information. The scalar field $F(\vec{x}, t)$ is defined as:

$$F(\vec{x}, t) = \begin{cases} 1, & \text{in the fluid;} \\ > 0, < 1, & \text{at the free surface} \\ 0 & \text{in the void} \end{cases}$$

The location of variables in the computational cell follows that of the Marker and Cell scheme (MAC). The x and y velocities are locate at the vertical and horizontal cell faces, and the pressure $p_{i,j}$ and VOF-function $F_{i,j}$ are locate at cell centers, see fig. 3.

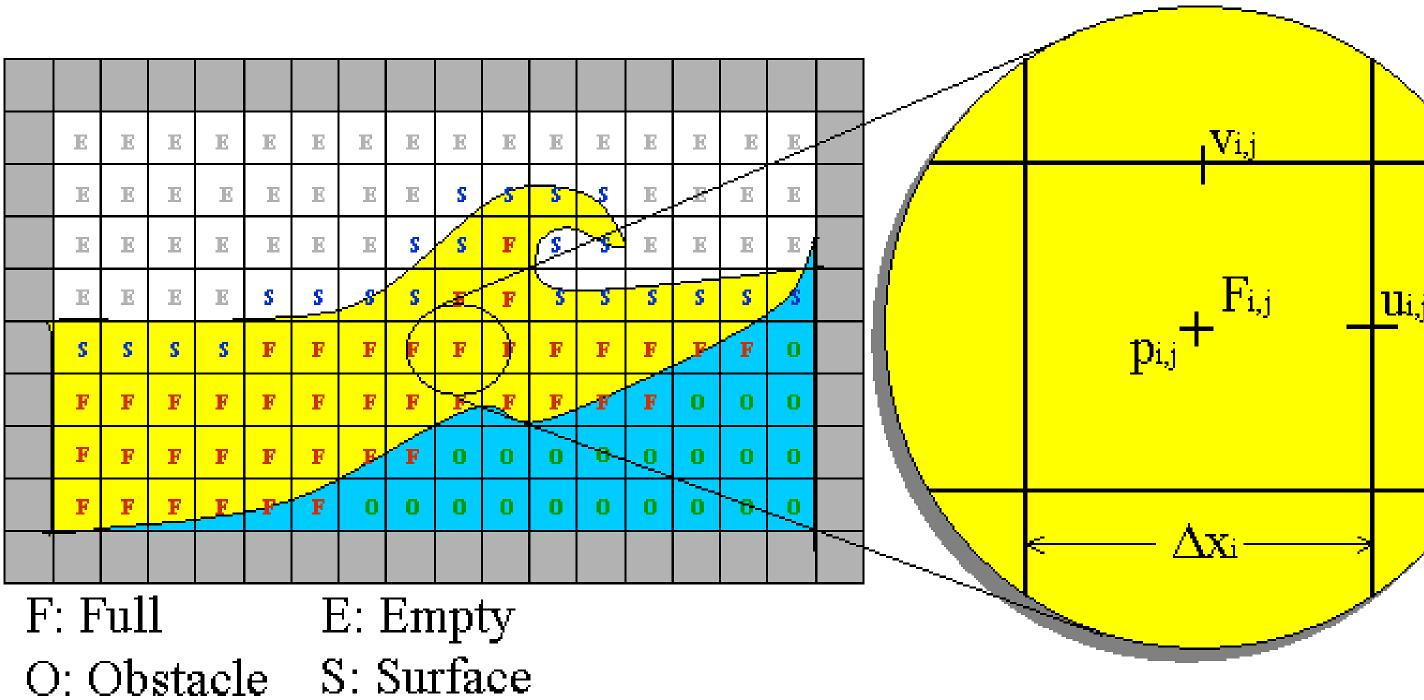

Figure 3. Location of variables in the VOF method.

This VOF solution has been incorporated to make comparison with SW. The differences are reasonable since this method solves the 2D N-S equations with the horizontal and vertical velocities being variable along the water column, while the SW assumes constant horizontal velocity. We conclude that VOF finite difference solution is a good candidate to benchmark "BM1". Results obtained by this method are shown in figs. 12 and 13.

4. Numerical Results

4.1 First Order

4.1.1 Sea Level

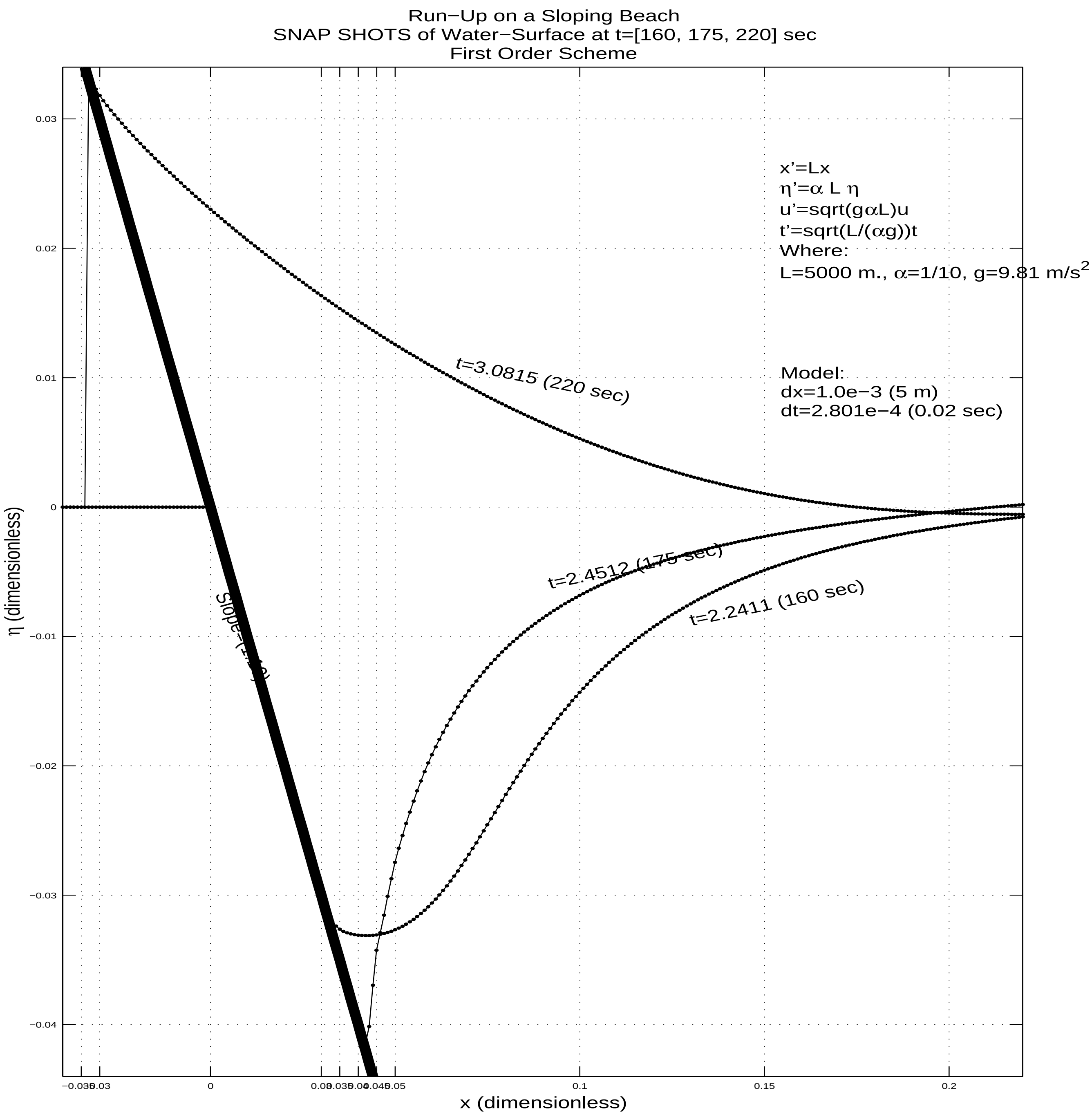

Figure 4.

4.1.2 Velocity

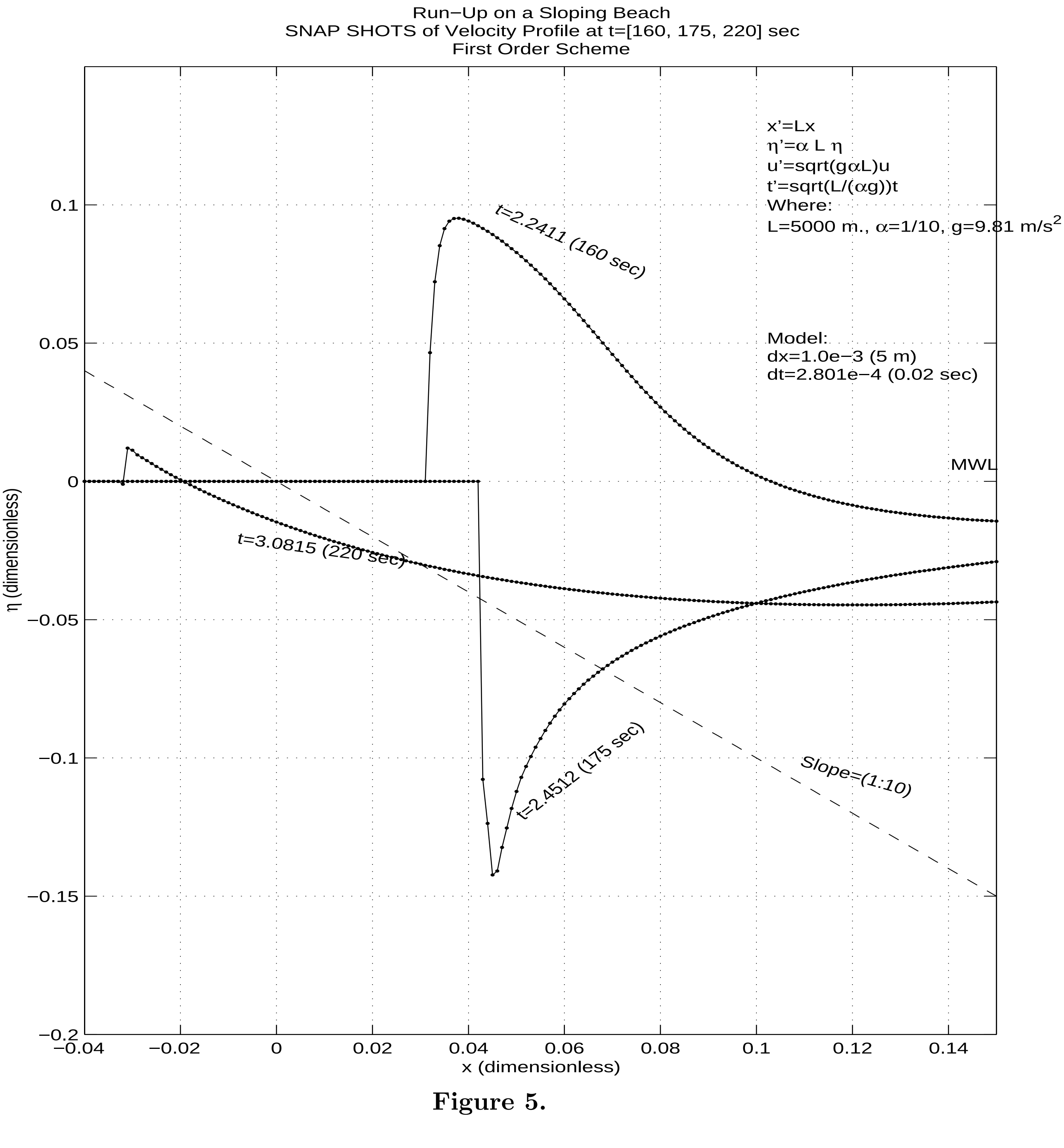

Figure 5.

4.1.3 Temporal and Spatial variation of the Sea Level

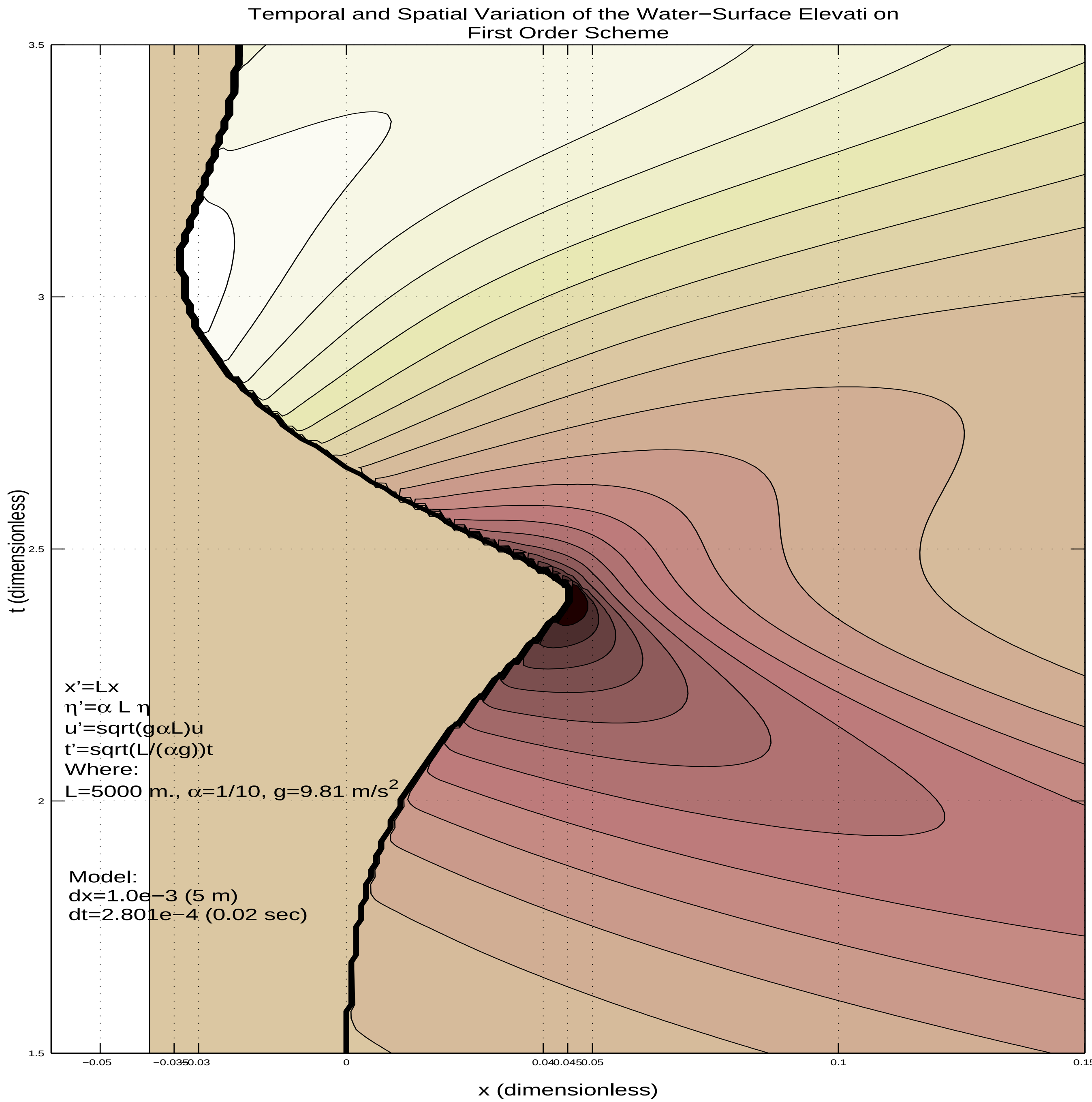

Figure 6.

4.1.4 Temporal and Spatial variation of the Velocity

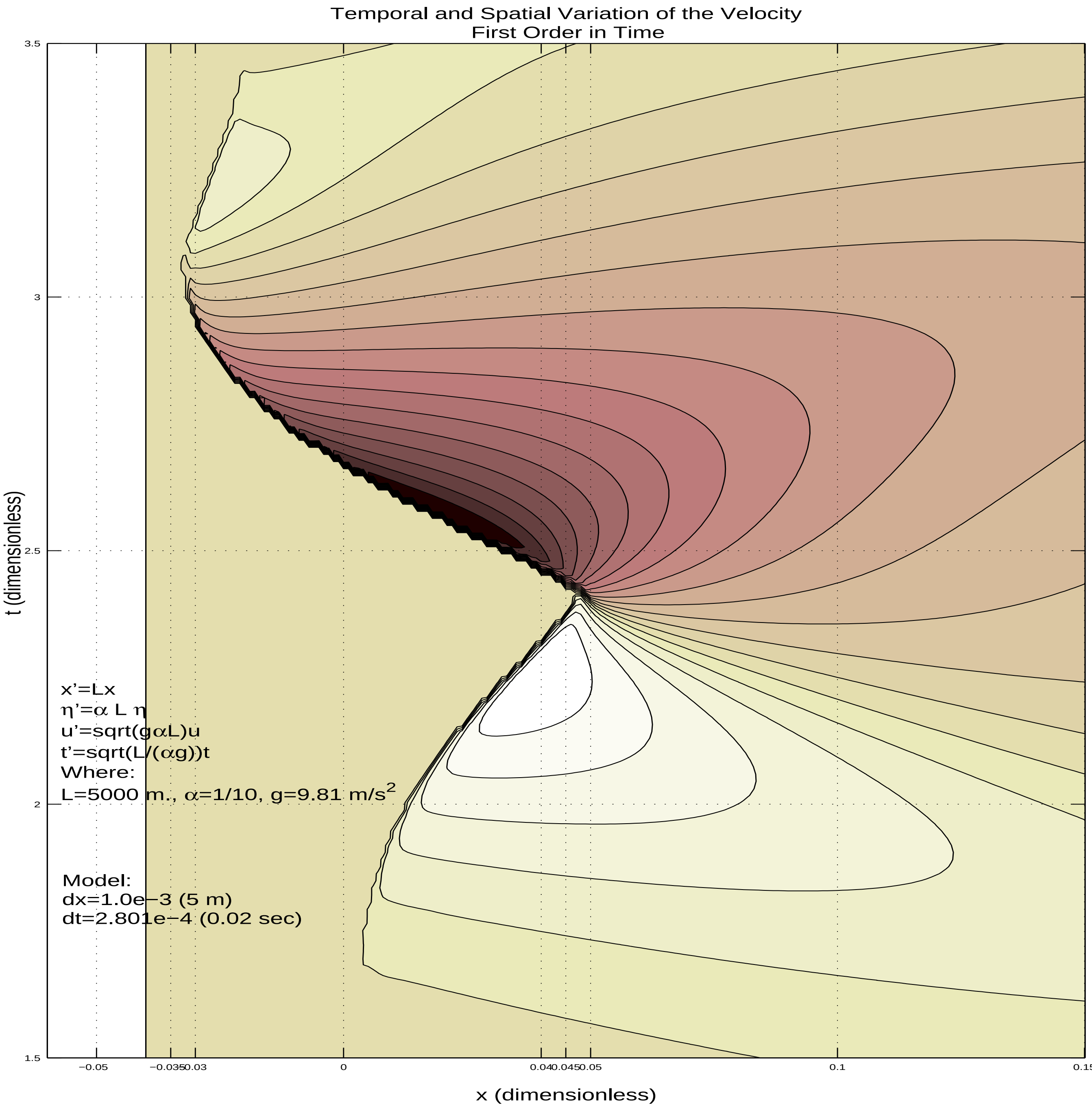

Figure 7.

4.2 Leap Frog Scheme

4.2.1 Sea Level

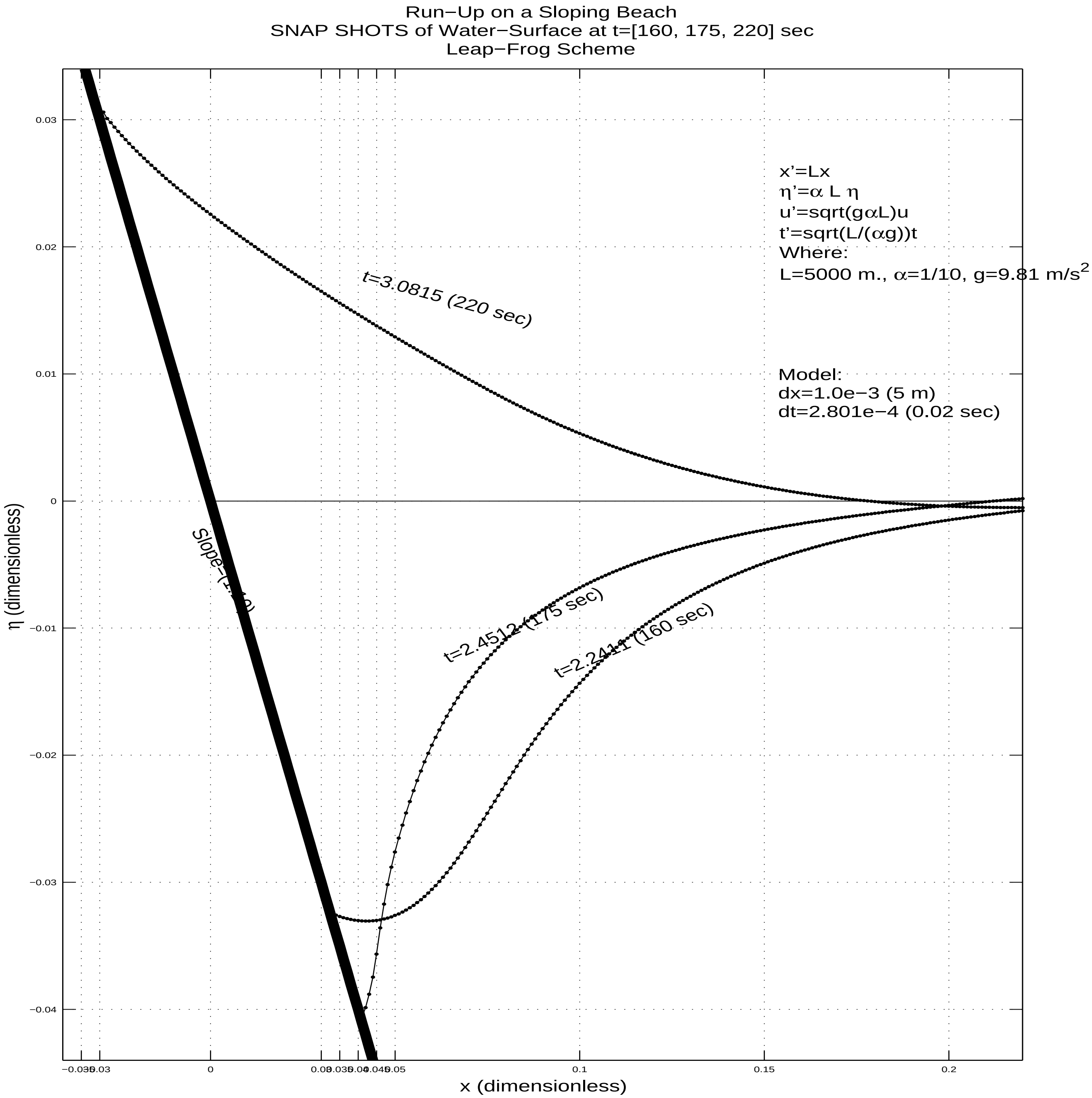

Figure 8.

4.2.2 Velocity

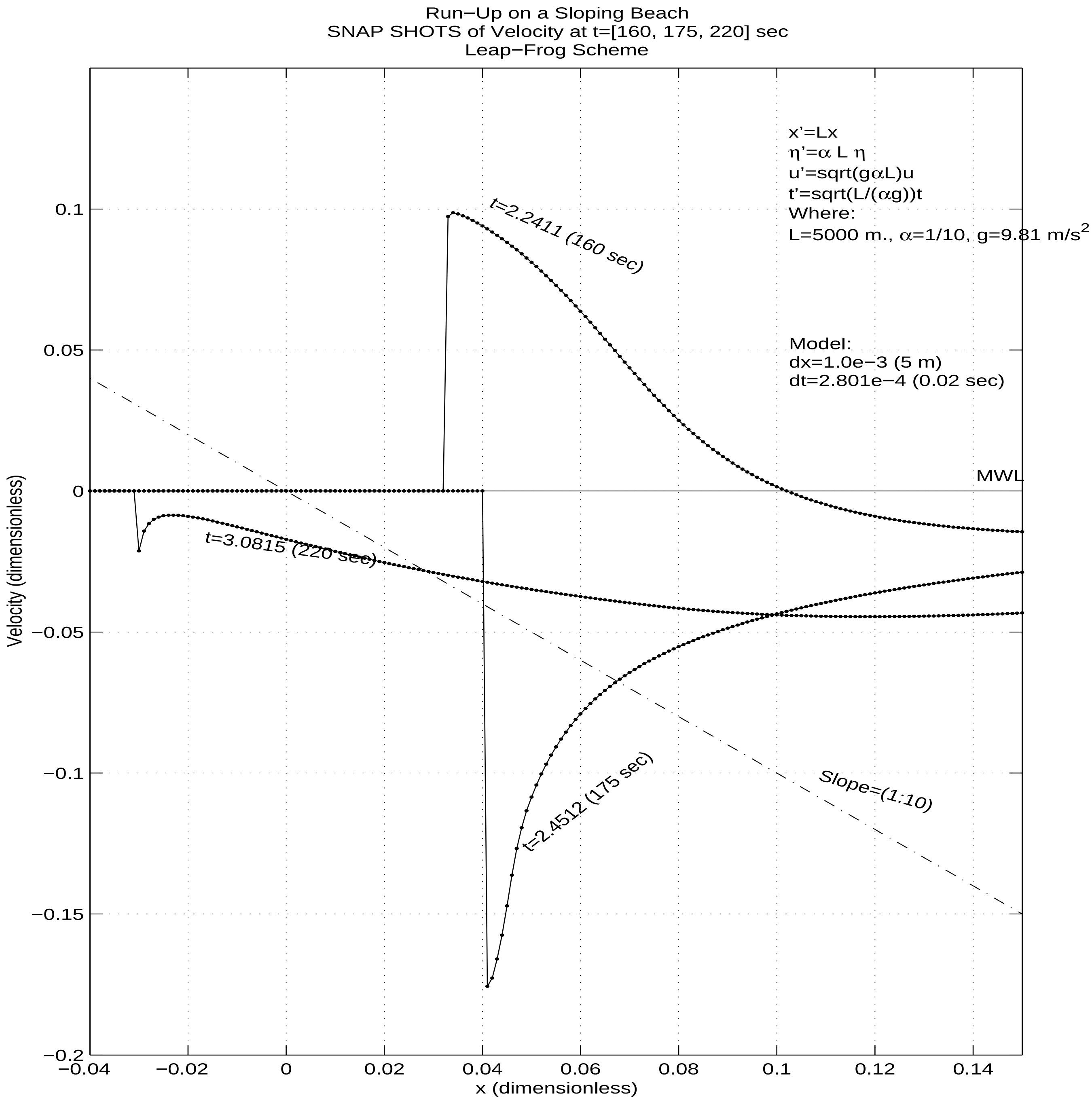

Figure 9.

4.2.3 Temporal and Spatial variation of the Sea Level

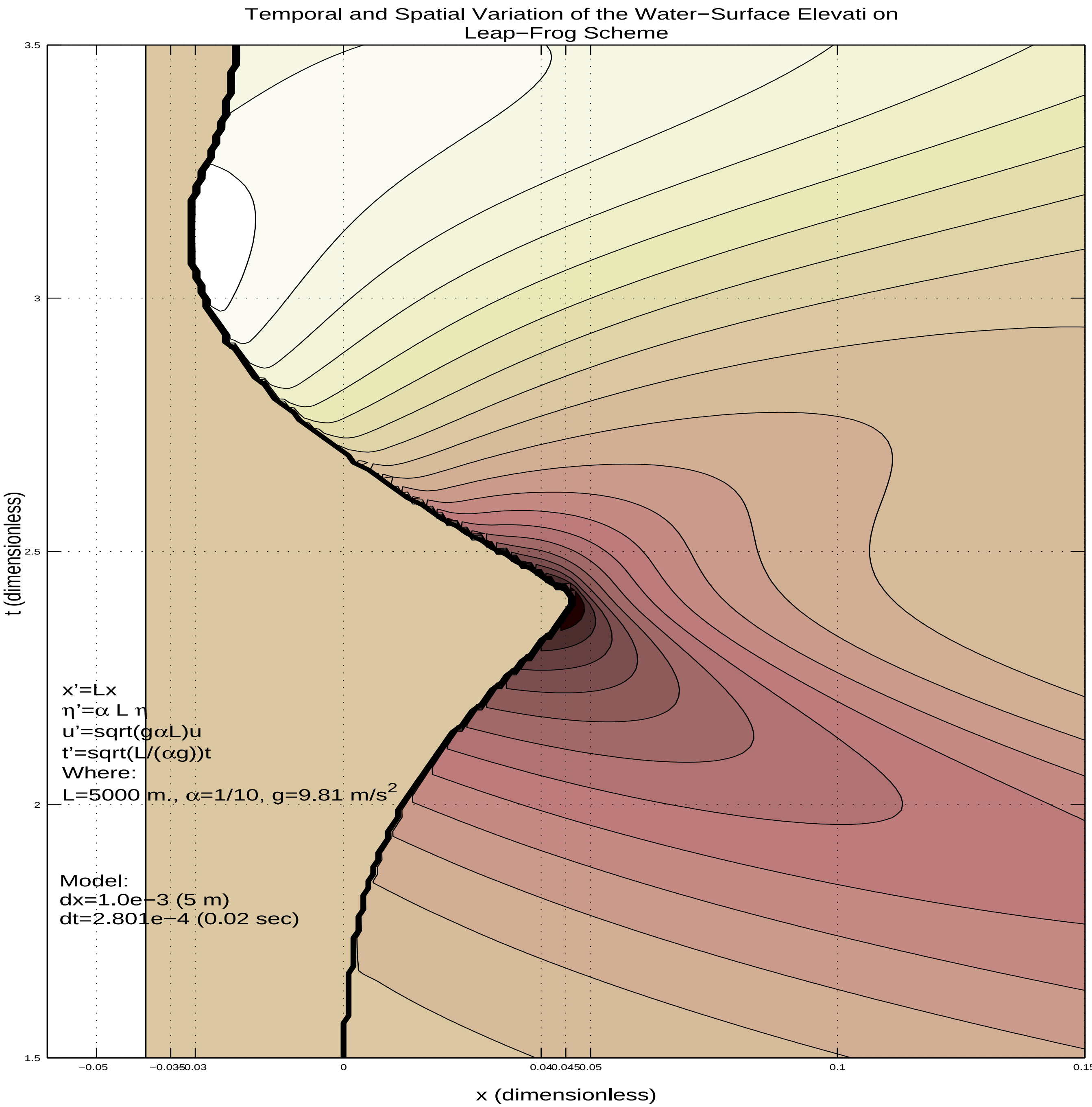

Figure 10.

4.2.4 Temporal and Spatial variation of the Velocity

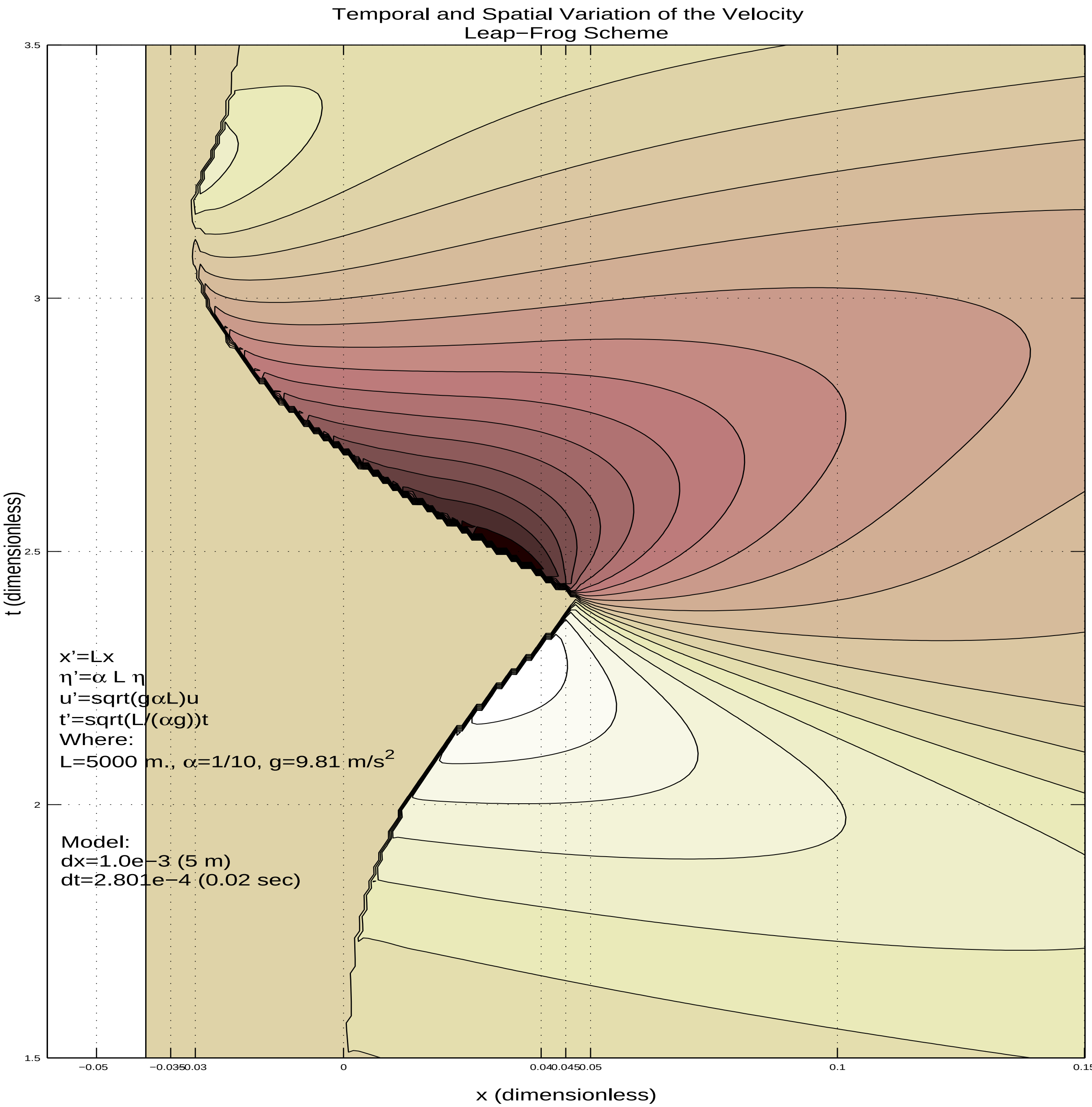

Figure 11.

4.3 VOF Method

4.3.1 Sea Level

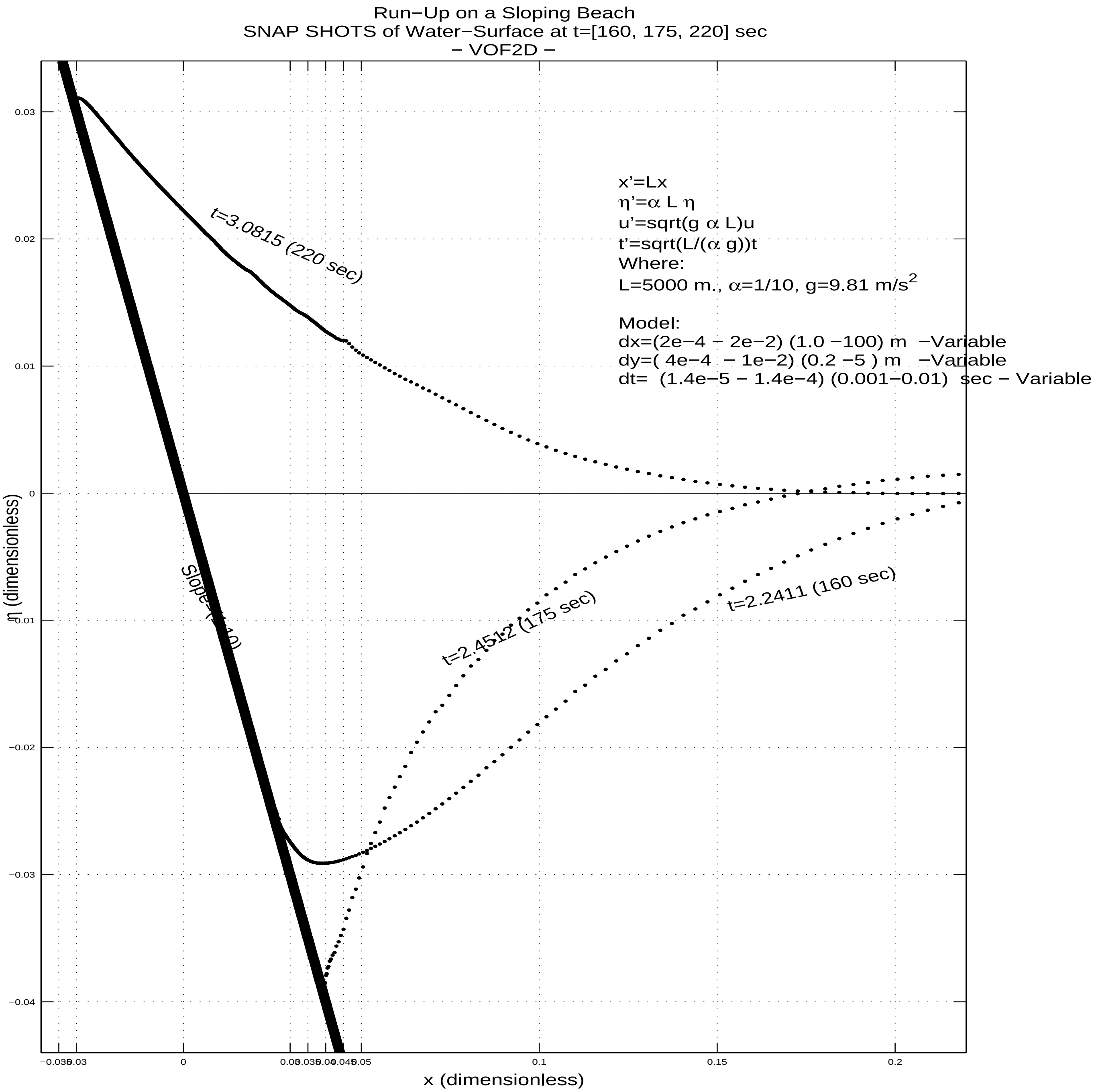

Figure 12.

4.3.2 Temporal and Spatial variation of the Sea Level

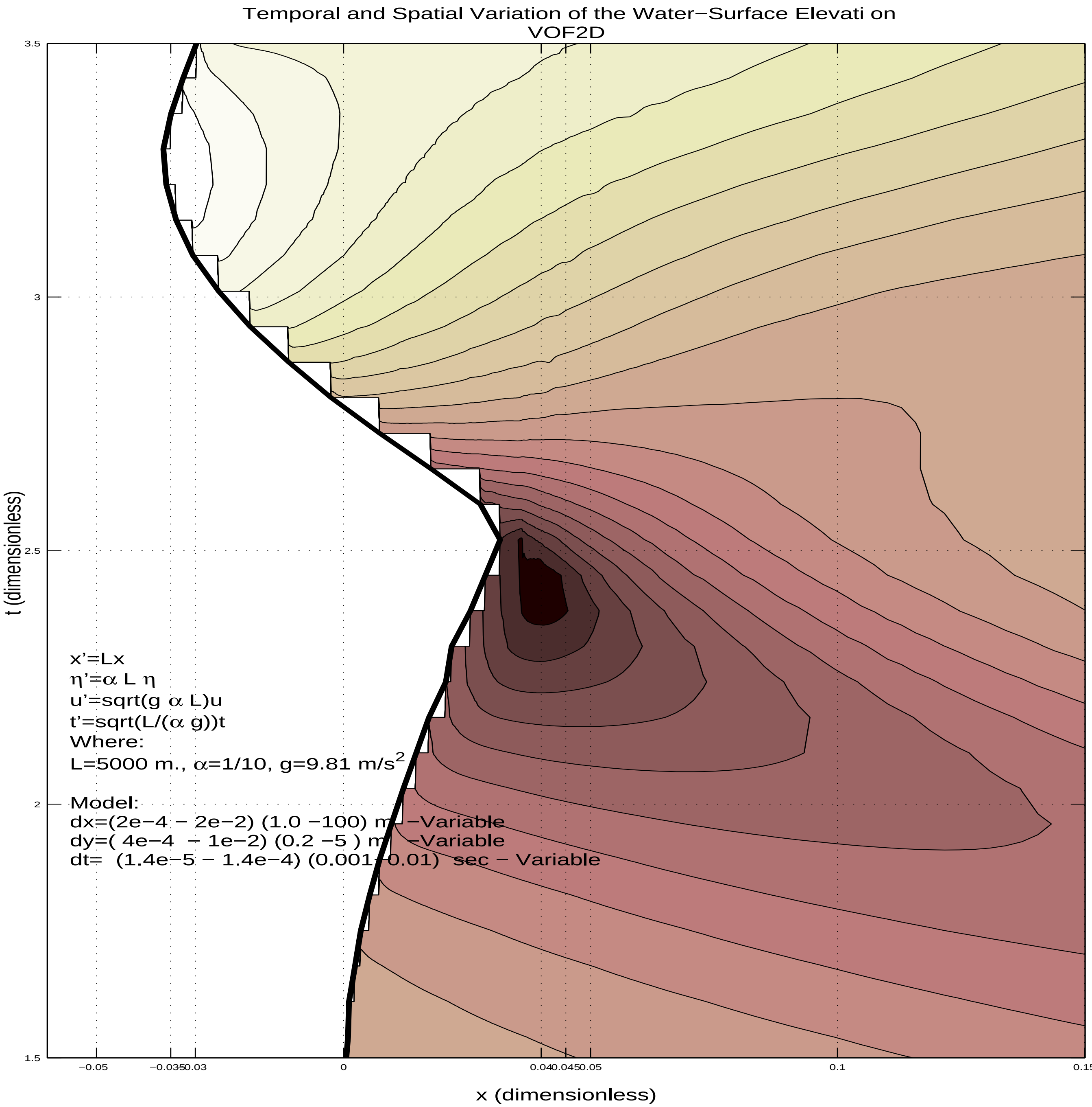

Figure 13.

5. Comparison

5.1 Temporal and Spatial variation of the shore Line

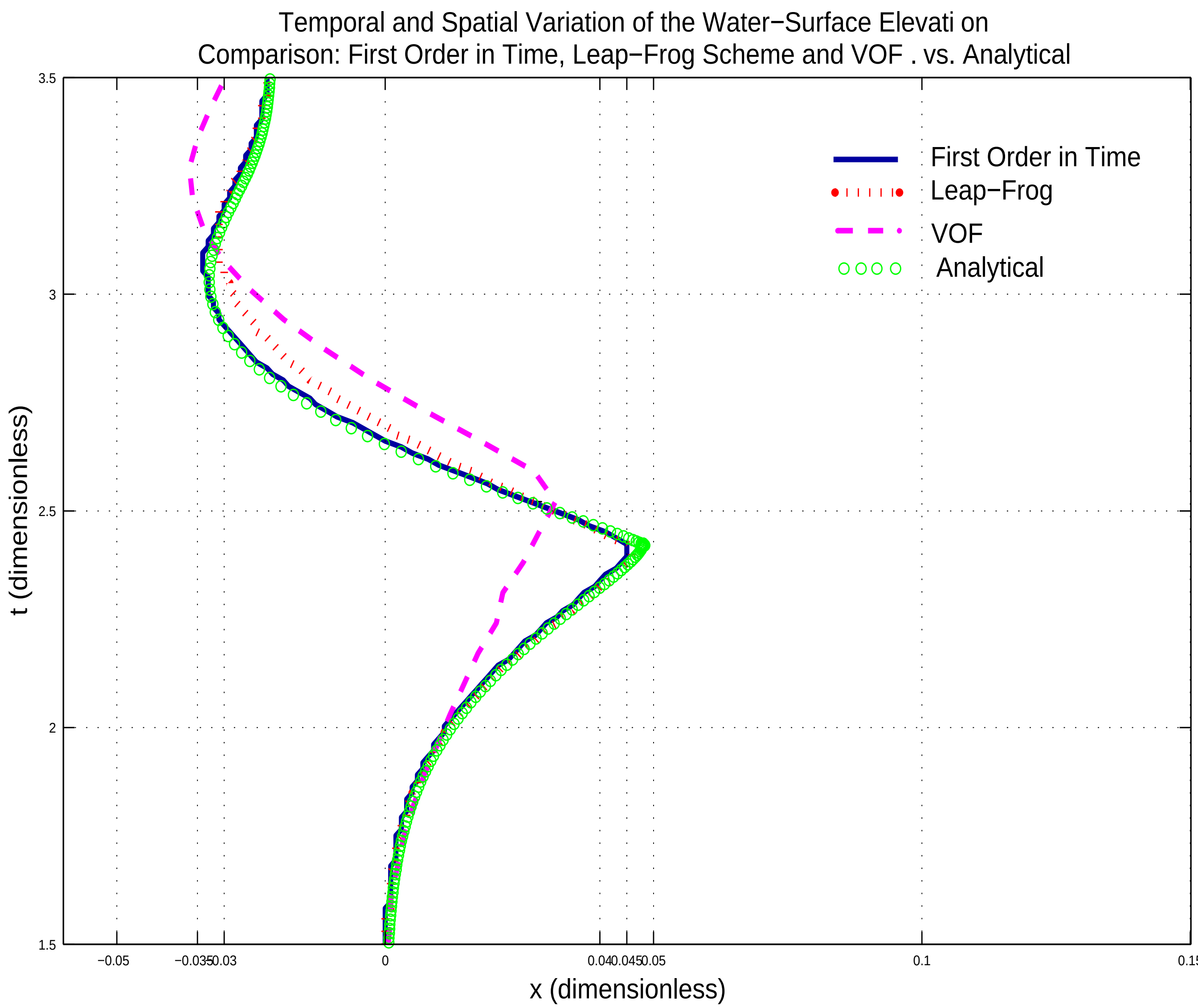

Figure 14.

5.2 Sea Level a)

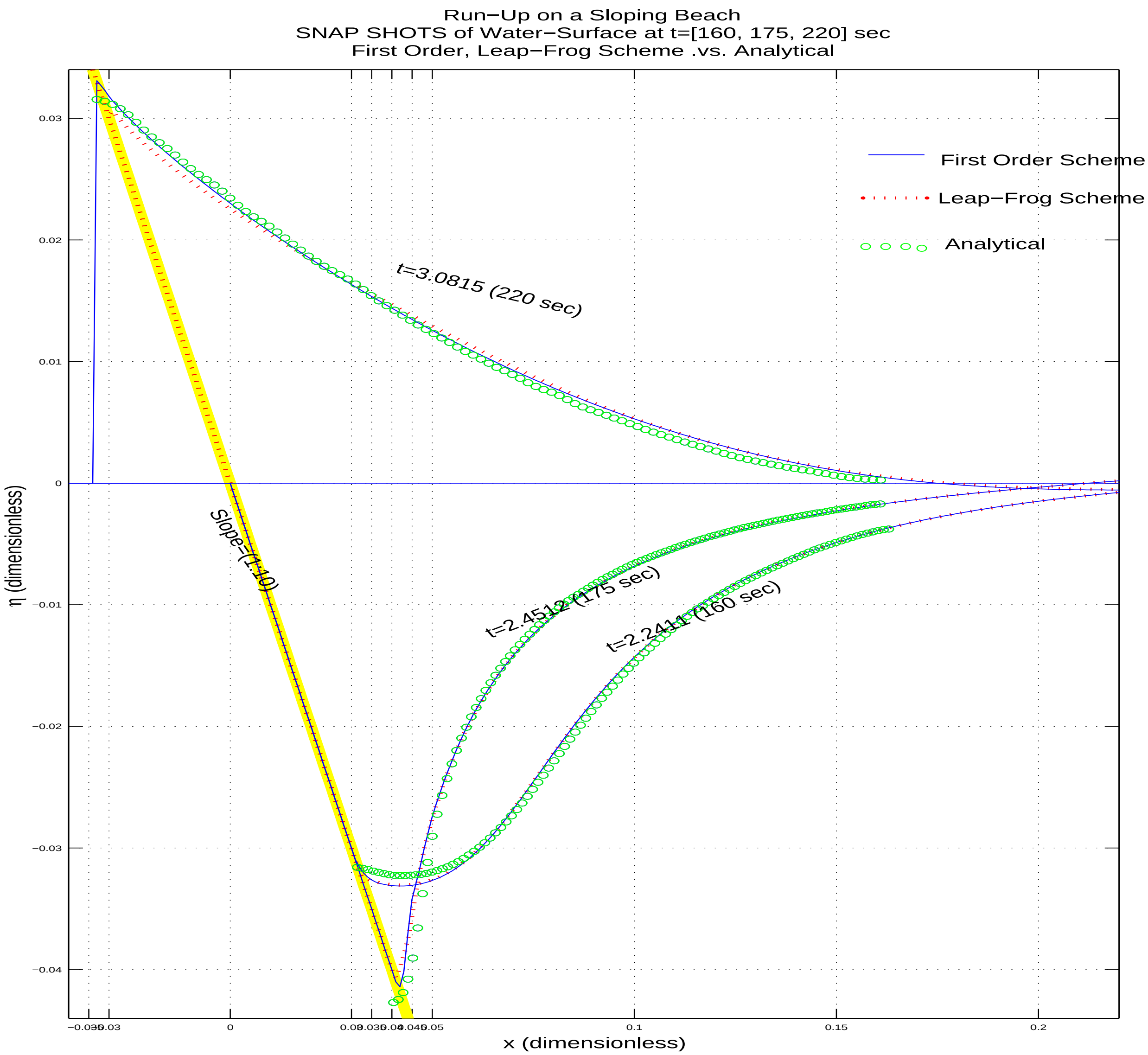

Figure 15.

5.3 Sea Level b)

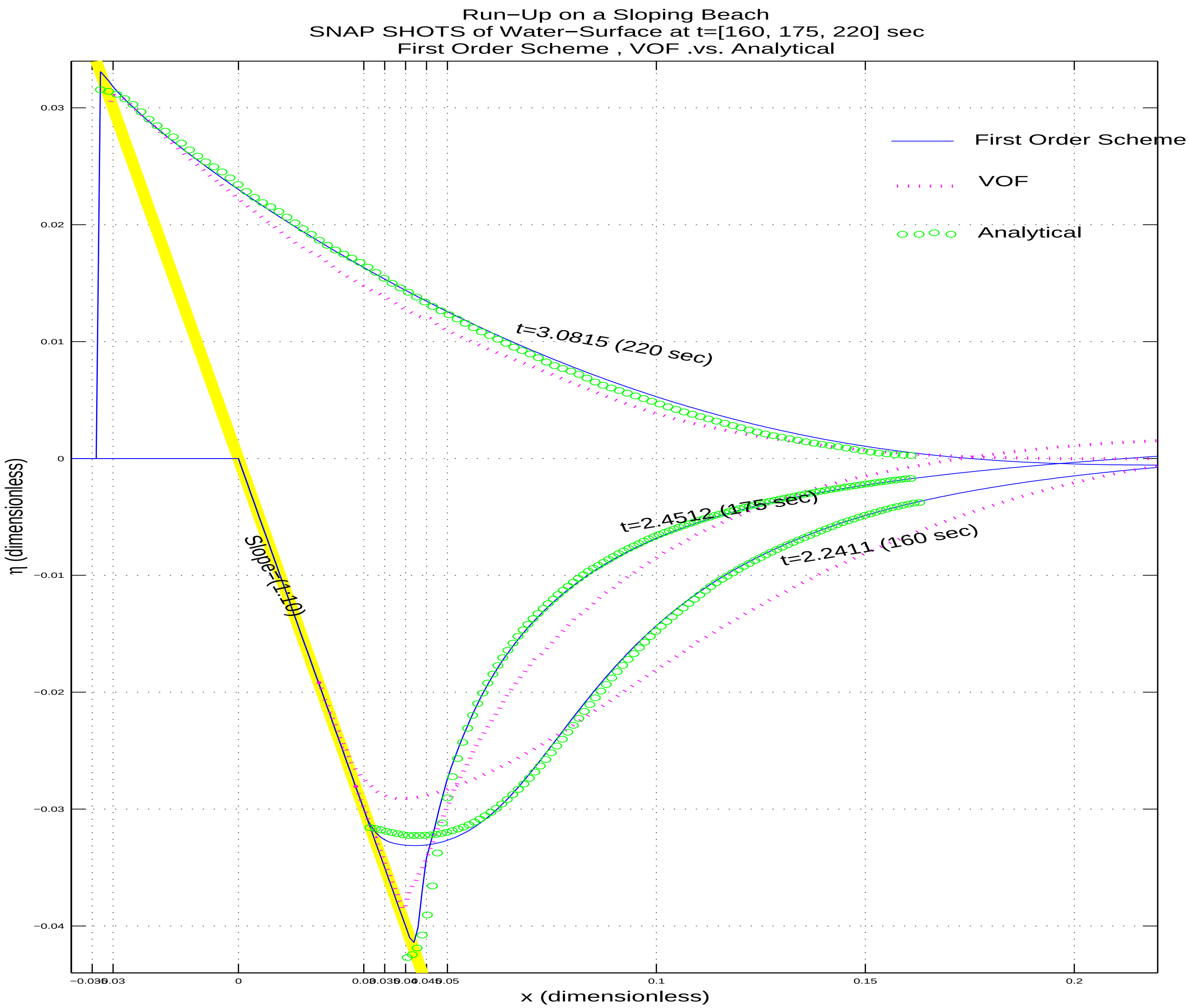

Figure 16.

5.4 Velocity Comparison

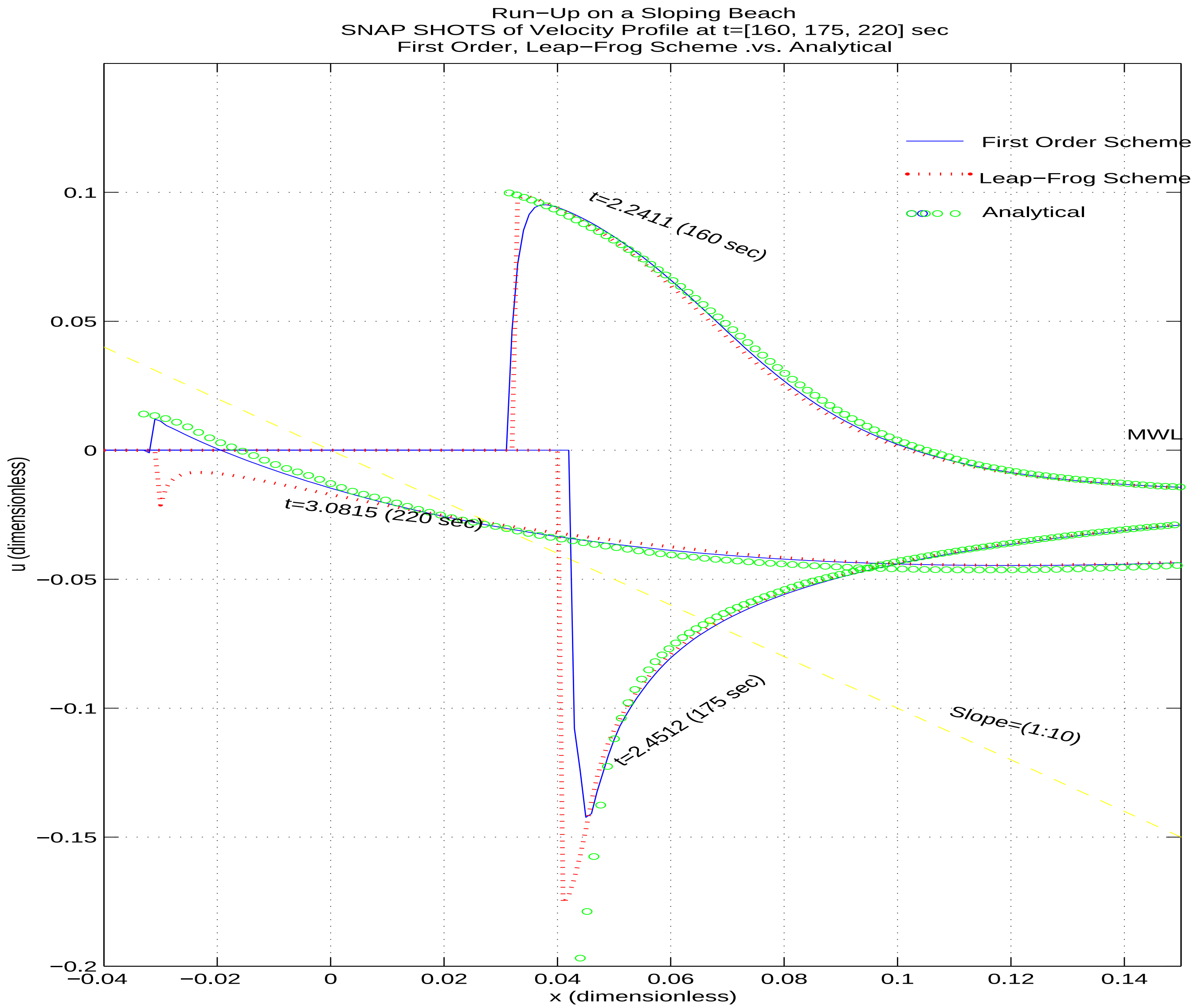

Figure 17.

6. Conclusion

Comparison of results (see figs. 14-17) indicates a good agreement between the first order method, second order method and in some extent with the VOF method. Shore line evolution is well predicted by the first order and second order method. For the first order method, assumption that the sea level of dry cell equals sea level of wet cell is physically reasonable. The extrapolation of the velocity of the immediate wet cell to the new wet cell facilitates runup and improves timing.

The VOF method gives a frame of reference to validate the SW theory. Some differences in the shore line evolution and timing are quite plausible, since VOF method allows vertical fluid acceleration while the SW theory does not.

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