THE THIRD INTERNATIONAL WORKSHOP ON LONG-WAVE RUNUP MODELS

June 17-18 2004
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June/2004

REPORT

BENCHMARK PROBLEM -3- (BM3)

Tsunami generation and runup due to a 2D landslide

1. Problem Description

The objective of this problem is to predict the free surface elevation and runup associated with translating a Gaussian shaped mass which is initially at the shoreline. In dimensional form, the seafloor can be described by:

$$\eta(x,t) = H(x) + \eta_o(x,t) \tag{1}$$

where:

$$H(x) = xtan(\beta) \tag{2}$$

$$\eta_o(x,t) = \delta exp\left(-\left(2\sqrt{\frac{x\mu^2}{\delta tan\beta}} - \sqrt{\frac{g}{\delta}}\mu t\right)^2\right) \tag{3}$$

and η_o is the slide thickness (in the paper by Liu, Lynett and Synolakis it is denoted as h_o), δ is the maximum vertical slide thickness, $\mu = \delta/L$ is the thickness-slide-length ratio, and β is the slope angle. Once in motion the mass moves at constant acceleration.

The following two setups are tested using our numerical models:

CASE A: $tan\beta/\mu = 10$. Where: $\beta = 5.7^o$, $\delta = 1m$ and $\mu = 0.01$,

CASE B: $tan\beta/\mu = 1$. Where: $\beta = 5.7^o$, $\delta = 1m$ and $\mu = 0.1$,

and the free surface location at selected times are determined according to:

CASE A: $\sqrt{\frac{g}{\delta}}\mu t = 0.1, 0.5, 1.0, 1.5$ (non-dimensional times)

CASE B: $\sqrt{\frac{g}{\delta}}\mu t = 0.5, 1.0, 2.5, 4.5$

Comparisons are made for each setup using the analytical solution posted on the web. For more information about the analytical solution, please refer to *Analytical solution for forced long waves on a sloping beach* by Liu, Lynett and Synolakis, *Journal of Fluid Mechanics*, **478**, 101-109, 2003.

2. Introduction

To Solve BM3, two approaches have been carried out:

a) First order approximation in time

b) Volume of Fluid Method (VOF)

Approach a) uses 1-D linear and non-linear shallow water wave theory, LSW and NLSW respectively. The finite difference solution of the equation of motion and the depth integrated continuity equation are solved on a staggered grid. This method possesses a second order approximation in space and one in time.

A 2-D VOF solution has been included to visualize differences between the shallow water theory (SW), analytical solution and VOF approach. The VOF method use the full nonlinear Navier-Stoke (N-S) equations and models transient and incompressible fluid flow with free surface. The finite difference solution of the incompressible N-S equations are obtained on a Eulerian rectilinear mesh.

3. Description of the Models

3.1 First Order Method

Equations of motion and continuity read,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \zeta}{\partial x} + \frac{1}{\rho D} ru|u| = 0 \tag{4}$$

$$\frac{\partial(\zeta - \eta_o)}{\partial t} + \frac{\partial uD}{\partial x} = 0 \tag{5}$$

where u is the vertically averaged velocity, ζ is the sea level, H is the mean water depth defined by eq. 2, $\eta_o(x,t)$ is the slide thickness defined by eq. 3, $D = (\zeta + H - \eta_o)$ is the total depth, r is the friction coefficient, ρ is the water density, and g is the gravity acceleration. The friction term appears in the equations but has been omitted in all cases.

The numerical solution of this system is usually searched by using one-time-level numerical sche
in equations (4

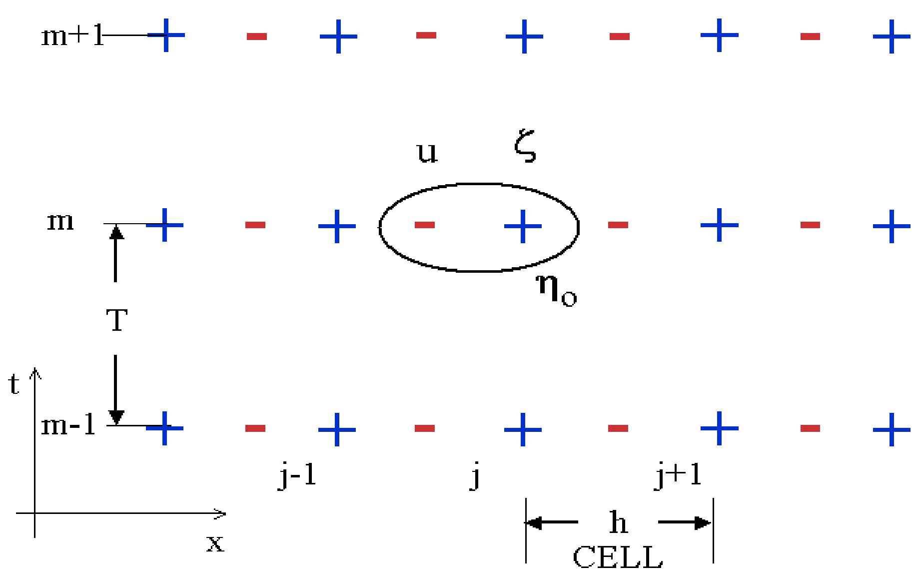

Figure 1. Time-space grid for the 1D problem.

The numerical scheme is constructed as follows

$$u_j^{m+1} = u_j^m - \frac{gT}{h}(\zeta_j^m - \zeta_{j-1}^m) - \frac{u_p^m T}{h}(u_j^m - u_{j-1}^m) - \frac{u_n^m T}{h}(u_{j+1}^m - u_j^m)$$

$$-\frac{2T}{\rho(D^m_{j-1}+D^m_j)}ru^m|u^m| \tag{5}$$

$$\zeta_j^{m+1}=\zeta_j^m-\frac{T}{h}(flux_{j+1}^{m+1}-flux_j^{m+1})+\eta_{oj}^{m+1}-\eta_{oj}^{m} \tag{6}$$

where:

$flux_j = u_p^{m+1}(\zeta_{j-1}^m-\eta_{oj-1}^{m})+u_n^{m+1}(\zeta_j^m-\eta_{oj}^{m})+u_j^{m+1}\frac{(H_j+H_{j-1})}{2}$
$u_p = 0.5*(u_j+|u_j|)$
$u_n = 0.5*(u_j-|u_j|)$

and

T is the time step, h is the space step. Index m and $j = 1, 2, 3, ...n-1, n$ stand for the time stepping and horizontal coordinate points respectively.

A simple runup condition, which is an extension of the previous BM1's runup condition is used. The following steps are taken when the dry point $j_{wet}-1$ is located to the left of the wet point j_{wet}:

$IF(\zeta^m(j_{wet}) > (-H(j_{wet}-1)+\eta_o(j_{wet}-1)))$ THEN

$$\zeta^m_{j_{wet}-1}=\zeta^m_{j_{wet}} \text{ and } u^m_{j_{wet}}=u^m_{j_{wet}+1}$$

A similar approach is used for the dry point located to the right of the wet point (Kowalik and Murty (1993b)).

Results obtained and comparisons with the analytical solution are depicted in figs. 3 and 4 for Cases A and B respectively. Agreements with the information posted are quite good. the Analytical solution does an excellent job of predicting wave runup and wave propagation for thin slides (Case A). For thick slides (Case B), omission of nonlinearity leads to disagreements in later stages of wave propagation. For a complete discussion about this topic see paper by Liu, Lynett and Synolakis, 2003.

3.2 VOF Method

The equation of continuity for incompressible fluid which includes conservation of mass due to a moving object into the control volume,

$$\nabla\cdot\vec{V}-\frac{1}{V_{ol}}\frac{\partial V_{ol}}{\partial t}=0 \tag{7}$$

and the momentum equation,

$$\frac{\partial\vec{V}}{\partial t}+\nabla\cdot(\vec{V}\vec{V})=-\frac{1}{\rho}\nabla p+\frac{1}{\rho}\cdot\tau+\vec{g} \tag{8}$$

are solved in the rectangular system of coordinates. There, $\vec{V}(x,y,t)$ is the velocity vector, V_{ol} is the moving object area or "volume" into the cell, ρ is the fluid density, p is the scalar pressure, τ is the viscous stress tensor, $\vec{g}$ is the acceleration due to gravity and t is the time.

Solution of the above equations is found using the two-step method, Harlow-Welch (1965), and Chorin (1968). The time discretization of the momentum equation is given by

$$\frac{\vec{V}^{m+1} - \vec{V}^m}{T} = -\nabla \cdot (\vec{V}\vec{V})^m - \frac{1}{\rho^m}\nabla p^{m+1} + \frac{1}{\rho^m} \cdot \tau^m + \vec{g} \tag{9}$$

and it is broken up into two steps as follow:

$$\frac{\tilde{\vec{V}} - \vec{V}^m}{T} = -\nabla \cdot (\vec{V}\vec{V})^m + \frac{1}{\rho^m} \cdot \tau^m + \vec{g} \tag{10}$$

$$\frac{\vec{V}^{m+1} - \tilde{\vec{V}}}{T} = -\frac{1}{\rho^m}\nabla p^{m+1} \tag{11}$$

$$\nabla \cdot \vec{V}^{m+1} - \left(\frac{1}{V_{ol}}\frac{\partial V_{ol}}{\partial t}\right)^{m+1} = 0 \tag{12}$$

Before any computation, $\frac{\partial V_{ol}}{\partial t}$ is known and can be calculated. It defines the amount of area the moving body is invading or releasing from the cell. The same amount of fluid has to be expelled or taken by the cell to fulfill the continuity equation.

In the first step, a velocity field $\tilde{\vec{V}}$ is computed from $\vec{V}^m$. In the second step, this velocity field is introduced into equation 11. Equation 11 and 12 can be combined into a single Poisson equation for the solution of the pressure as,

$$\nabla \cdot [\frac{1}{\rho^m}\nabla p^{m+1}] = \frac{\nabla \cdot \tilde{\vec{V}}^{m+1} - \left(\frac{1}{V_{ol}}\frac{\partial V_{ol}}{\partial t}\right)^{m+1}}{T}. \tag{13}$$

The free surface of the fluid is described with the discrete volume-of-fluid (VOF) method. The VOF method, pioneered by Hirt and Nichols (1975) is a powerful tool that enables a finite difference representation of the free surface and interfaces that are arbitrarily oriented with respect to the computational grid. This method has been used for wave generation due to landslide, (see works done by Mader (1999) and Heinrich (1991 and 1992)).

The VOF function is advected as a Lagrangian invariant, propagating according to

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + (\vec{V} \cdot \nabla)F = 0 \tag{14}$$

and is the only free surface information. The scalar field $F(\vec{x}, t)$ is defined as:

$$F(\vec{x}, t) = \begin{cases} 1, & \text{in the fluid;} \\ > 0, < 1, & \text{at the free surface} \\ 0 & \text{in the void} \end{cases}$$

The location of variables in the computational cell follows that of the Marker and Cell scheme (MAC). The x and y velocities are located at the vertical and horizontal cell faces, and the pressure $p_{i,j}$ and VOF-function $F_{i,j}$ are located at cell centers; see fig. 2.

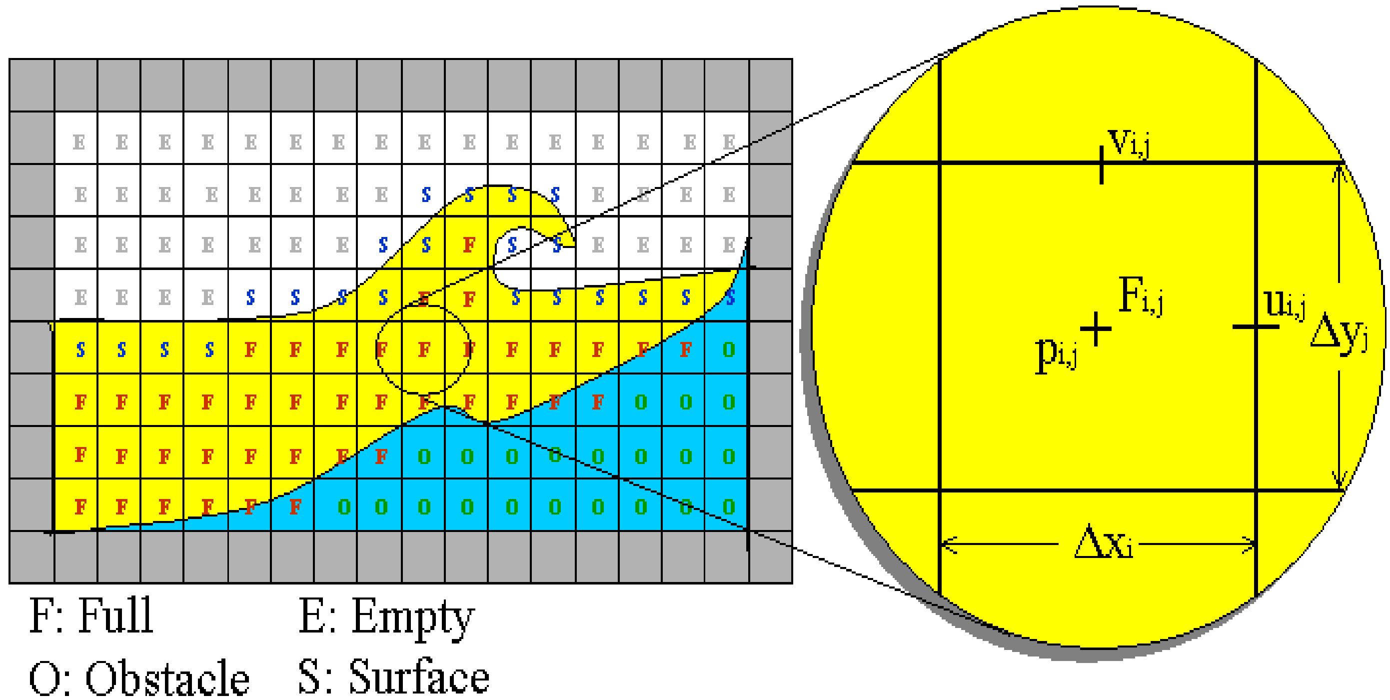

Figure 2. Location of variables in the VOF method.

We used VOF to compare with NLSW for Case B, see fig. 5. Case B has been chosen because it offers a more interesting vertical acceleration than case A. As can be gleaned from fig. 5, VOF's results agreed very with NLSW approach. Some differences are visualized, like the wave skewness and location of the shore line, see fig. 6. It is important to mention that VOF method solves the 2-D N-S equations with the horizontal and vertical velocities being variable along the water column, while the SW assumes constant horizontal velocity. The SW approaches will miss the physics caused by the vertical acceleration. We think that the VOF solution is ideally suited for this case, where relatively high vertical acceleration occurs. In this particular case, VOF predicts a wave a little more elongated, skewed and less tall than the NLSW theory for later time (i.e. t=2.5 and 4.5). All numerical models and analytical solution agreed very well at earlier time (i.e. t=0.5 and t=1).

4. Numerical Results and Comparison with analytical solution

4.1 First Order

4.1.1 CASE A

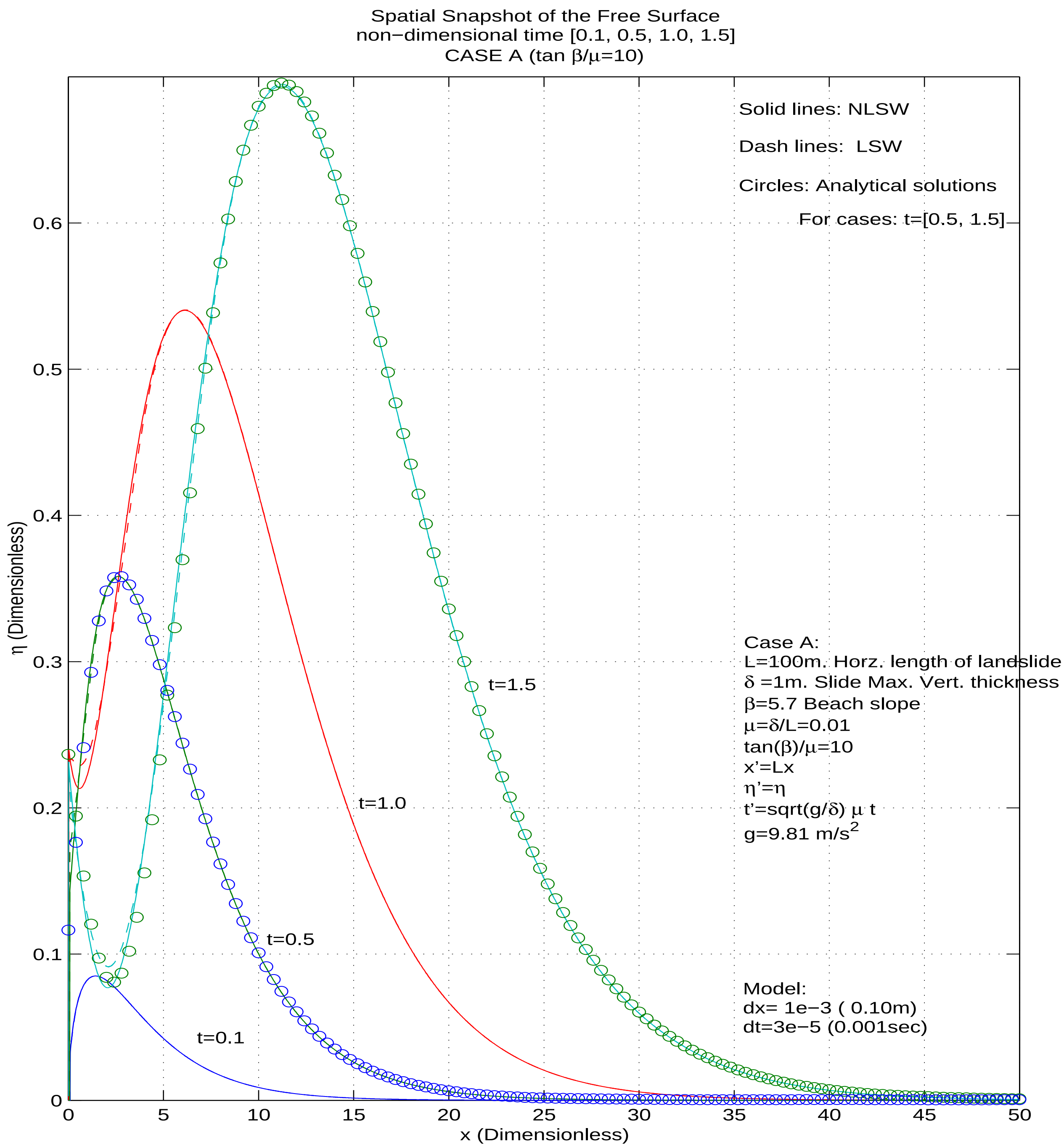

Figure 3.

4.1.2 CASE B

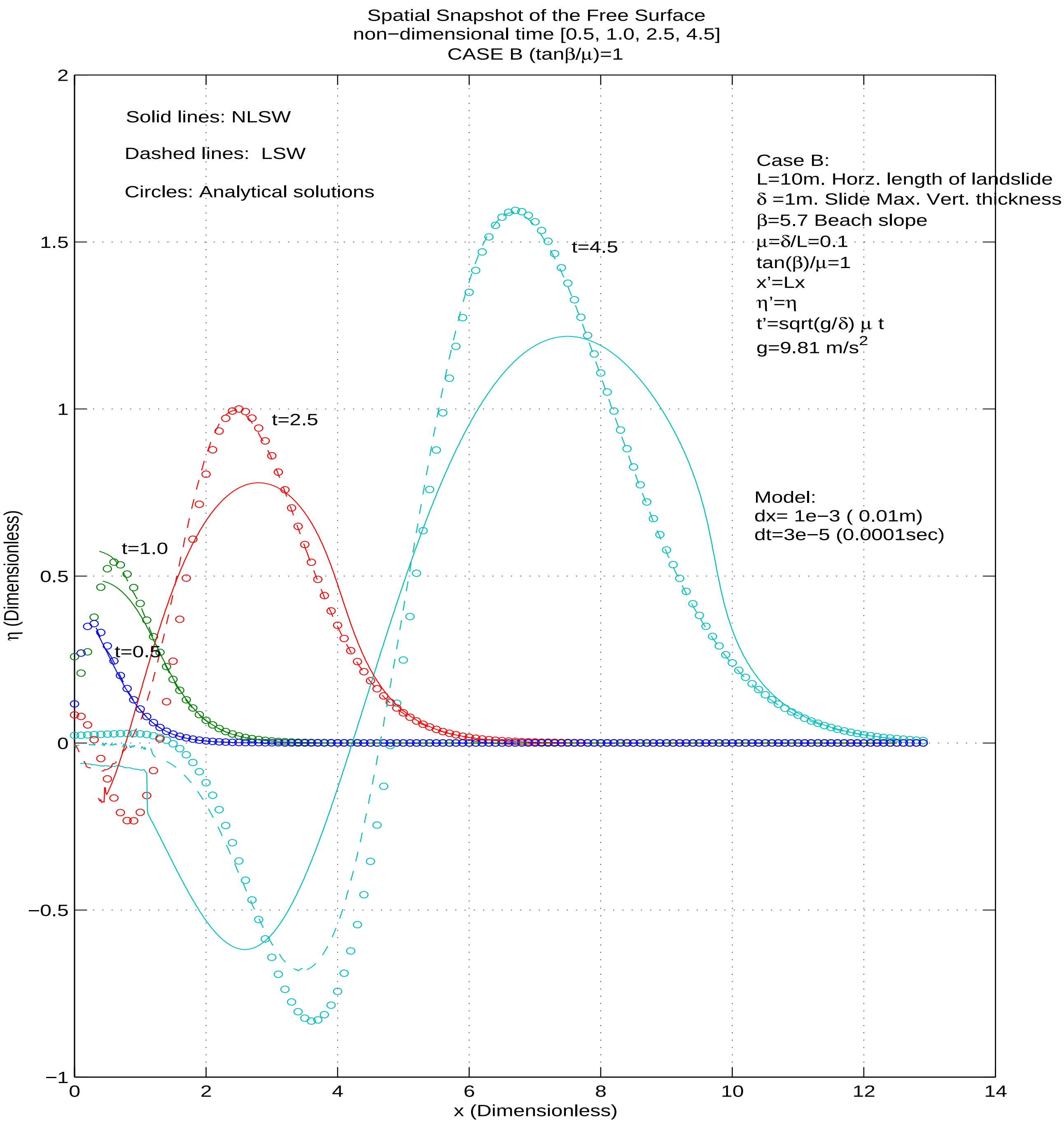

Figure 4.

4.2 VOF Method

4.2.1 CASE B -Free surface Snapshot-

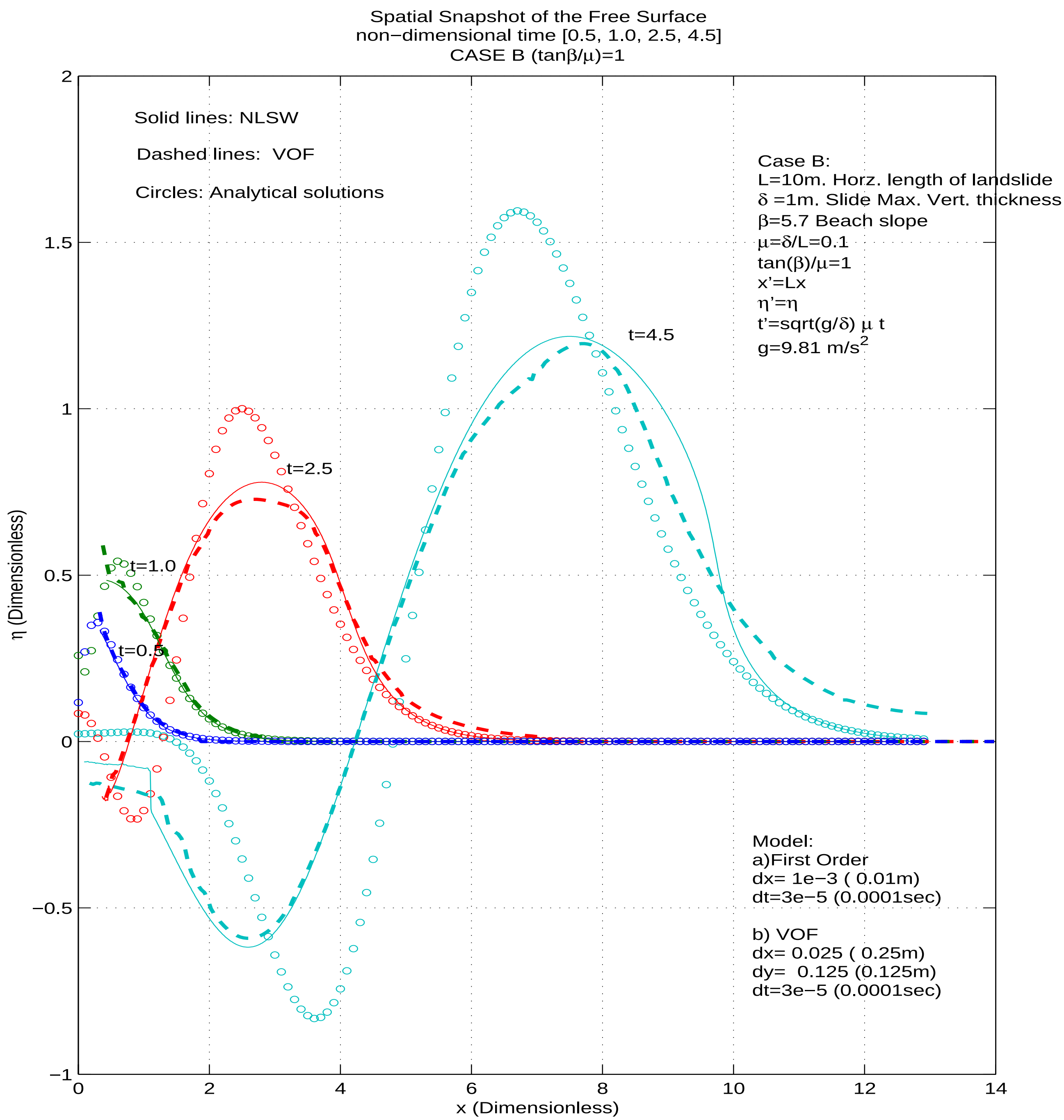

Figure 5.

4.2.2 CASE B -Shoreline Location-

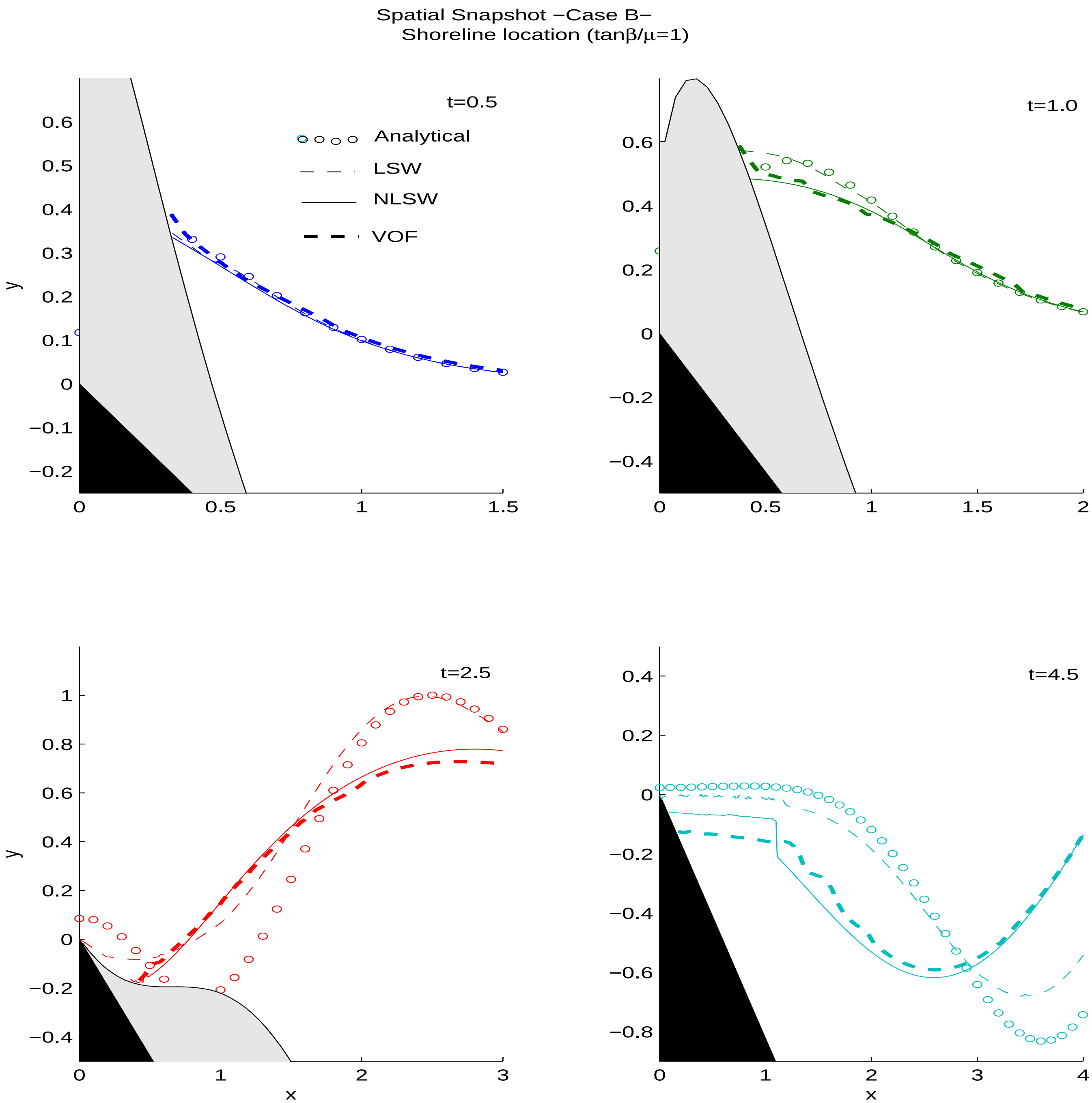

Figure 6.

5. Conclusion

To Solve BM3, two approaches have been carried out: a)First order approximation in time, using NLSW and LSW theory, and b)Volume of Fluid Method. Agreements for the LSW and NLSW versus analytical solution are quite good. Analytical solution does an excellent job in predicting wave runup and wave propagation for thin slides. For thick slides, omission of the nonlinear term, leads to differences in solutions, which become stronger in time.

A VOF solution is presented for case B in order to compare with the NLSW. The comparison shows some rasonable differences since this method solves the 2-D N-S equations with the horizontal and vertical velocities being variable along the water column, while the SW assumes constant horizontal velocity. Results obtained by this method predict a more realistic wave evolution and shoreline location.

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